MAT1505-03/04 19F Quiz 5 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

You may use technology to evaluate the necessary antiderivatives, but you must use the limit definition which allows evaluation of the improper integrals. Check your work by directly evaluating these integrals with technology.

1. $\displaystyle\int_2^3 \frac{1}{(x-2)^{\frac{2}{3}}}\,dx$

This is a quiz on limits, not integration techniques. Why would anyone waste time on integration by parts for 2, and the u-sub in 1 is obvious— unnecessary to go thru change of variable steps.

2.a) $\displaystyle\int_0^\infty x\,e^{-\frac{x}{3}}\,dx$

b) What percent of this area under the graph out to infinity is represented by the integral from $x = 0$ to the location of the maximum value of the integrand? Use calculus to derive the location of the maximum value before proceeding.

▶ solution

simple u-sub (obvious)

① $\displaystyle\int (x-2)^{-2/3}dx = \frac{(x-2)^{1/3}}{1/3} + C = 3(x-2)^{1/3} + C$

rightsided limit, undefined at $x=2$! limit from inside interval

$$\int_2^3 (x-2)^{-2/3}dx = \lim_{t\to 2+}\int_t^3 (x-2)^{-2/3}dx = \lim_{t\to 2+} 3(x-2)^{1/3}\Big|_t^3$$

$$= \lim_{t\to 2+}\big(3\underbrace{(3-2)^{1/3}}_{1} - 3\underbrace{(t-2)^{1/3}}_{\to 0}\big) = \boxed{3}$$

② a) $\displaystyle\int x e^{-\frac{x}{3}}dx \overset{\text{Maple}}{=} -3(x+3)e^{-x/3}$

$$\int_0^\infty x e^{-\frac{x}{3}}dx = \lim_{t\to\infty}\int_0^t x e^{-x/3}dx = \lim_{t\to\infty} -3(x+3)e^{-x/3}\Big|_0^t$$

$$= \lim_{t\to\infty}\big(\underbrace{-3(t+3)e^{-t/3}}_{} + \underbrace{3(0+3)e^0}_{9}\big) = \boxed{9}$$

$\dfrac{t+3}{e^{t/3}} \to 0$ [$\frac{\infty}{\infty}$ l'Hopital's rule, or growing exponentials beat any powers]

b) $\dfrac{d}{dx}\left(x e^{-x/3}\right) = 1e^{-x/3} + x e^{-x/3}(-1/3) = \underbrace{e^{-x/3}}_{\neq 0}\left(1-\frac{x}{3}\right) = 0$

$\neq 0 \to 1-\frac{x}{3} = 0 \to \frac{x}{3} = 1 \to x = 3$ } steps to "solve" for x

$$\int_0^3 x e^{-\frac{x}{3}}dx = -3(x+3)e^{-x/3}\Big|_0^3 = -3(3+3)e^{-3/3} + 3(0+3) = 9 - 18e^{-1}$$

$$\frac{\int_0^3 x e^{-x/3}dx}{\int_0^\infty x e^{-x/3}dx} = \frac{9-18e^{-1}}{9} = 1-2e^{-1} \approx .26424 \longrightarrow \boxed{\text{about } 26.4\%}\ \boxed{\text{or } 26\%}$$

percents are usually for interpretation so precision not usually useful