MAT1505-03/04 19F Quiz 3 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. The region R is bounded by the curves $y = x^2$ and $y = x^3$ over the interval $0 \le x \le 1$.

a) Make a diagram of this region R in the first quadrant, labeling axes and curves appropriately: show a typical vertical cross-section labeling the bullet endpoints by the starting and stopping values of y, namely $y = \ldots$, etc, [equations are needed to specify curves in the plane] and "shade in" this region with parallel equally spaced linear cross-sections.

b) Rotate this region R around the axis $y = 0$. Make a new diagram showing the reflection of the region across the axis resulting from this rotation and indicate in your new diagram at the typical cross-section from part a) the inner and outer radii corresponding to rotating those bullet point endpoints about the axis.

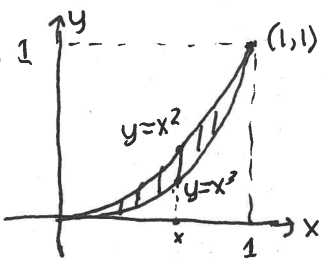
Write down an integral V_1 (with simplified integrand) representing the volume of the resulting solid of revolution.

c) Next rotate this same region R around the axis $x = 2$. Repeat the instructions of b) for the corresponding volume V_2, making a new expanded diagram again indicating the inner and outer radii at a typical linear cross-section perpendicular to the symmetry axis.

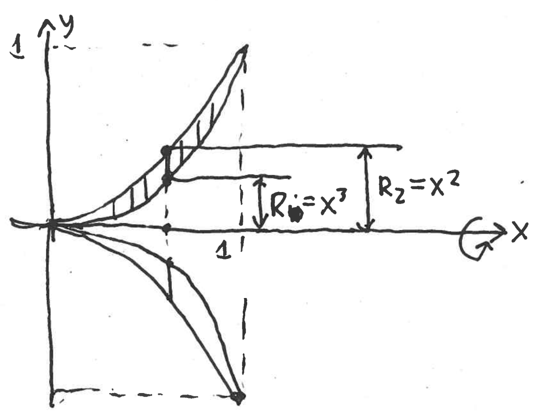
d) Evaluate V_1 and V_2 exactly with technology, and then each to 6 decimal places. [Check: $\frac{V_1}{V_2} = \frac{12}{49}$].

▶ solution

①a)



b)



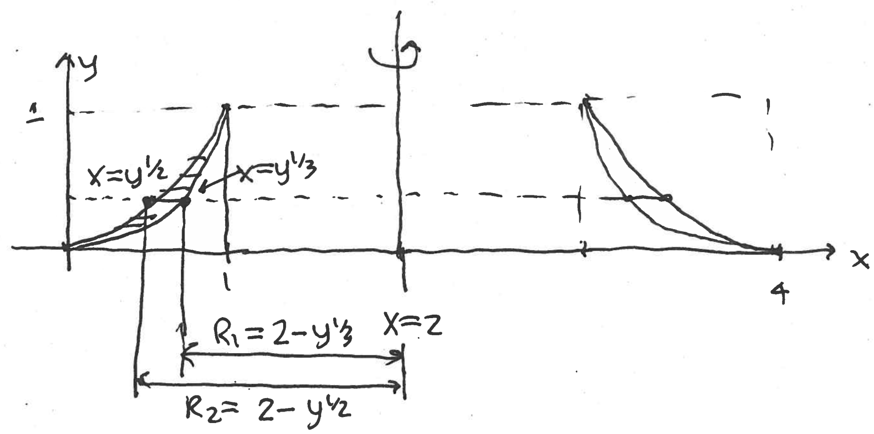
$$V_1 = \int_0^1 \pi\,(R_2^2 - R_1^2)\,dx$$

$$= \int_0^1 \pi\,((x^2)^2 - (x^3)^2)\,dx$$

$$= \boxed{\int_0^1 \pi\,(x^4 - x^6)\,dx}$$

d) $= \boxed{\frac{2\pi}{35} \approx 0.1795195802 \approx 0.179520}$

c)



$$V_2 = \int_0^1 \pi\,(R_2^2 - R_1^2)\,dy$$

$$= \int_0^1 \pi\,((2 - y^{1/2})^2 - (2 - y^{1/3})^2)\,dy$$

$$= \int_0^1 \pi\,(4 - 4y^{1/2} + y - (4 - 4y^{1/3} + y^{2/3}))\,dy$$

$$= \boxed{\int_0^1 \pi\,(-4y^{1/2} + y + 4y^{1/3} - y^{2/3})\,dy}$$

d) $= \boxed{\frac{7\pi}{30} \approx 0.7330382858 \approx 0.733038}$

check: $\frac{V_1}{V_2} = \frac{2\pi/35}{7\pi/30} = \frac{2}{7 \cdot 5} \cdot \frac{5 \cdot 6}{7} = \frac{12}{49}$