MAT1505-03/04 19F Quiz 2 Print Name (Last, First) ________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

a) When a body is falling from rest near the Earth's surface subject to air resistance, the time it takes to reach half the so called "terminal velocity" $v_{terminal}$ (speed! downward velocity) is

$$T_{half} = \int_0^{\frac{v_{terminal}}{2}} \frac{1}{g\cdot\left(1+\left(\frac{v}{v_{terminal}}\right)^2\right)}\,dv\,,$$

where g is the gravitational constant near the Earth's surface, ρ is a positive coefficient of friction to model the drag of air resistance of a freely falling body, and $v_{terminal} = \sqrt{\frac{g}{\rho}}$ is the positive terminal velocity.

do not use this — keep results in terms of $V_{terminal}$

a) Perform the change of variable to the dimensionless velocity variable $u = \frac{v}{v_{terminal}}$ to rewrite this definite integral in the form $K\int_{u_1}^{u_2} f(u)\,du$, namely as an overall constant factor containing these parameters (and carrying the dimension of the result) times a purely numerical integral without any parameters in it, i.e., just a pure number coefficient.

b) Evaluate the new integral exactly using Maple and then approximately to 4 significant figures. Does this agree with what Maple evaluates for the original integral?

c) Dimensionally if velocity has units of length per sec, and the gravitational constant which is an acceleration has units of length per sec squared, what are the units of their ratio? This should confirm that the units of the constant K carrying the dimensions in $T_{terminal}$ are correct!

d) How long (T_{99}) would it take to reach 99 percent of the terminal velocity? Round off your coefficient to 4 significant figures. Notice that the quantity K provides the natrual tickmark interval for all such similar questions about this timing.

(there is no mention of seconds or minutes for time here — so no need to introduce them — these determine the values of g and ρ)

a) $u = V/V_t \quad du = dV/V_t$

$dV = V_t\,du$

$V = 0 \to u = 0$

$V = \frac{V_t}{2} \to u = \frac{V_t/2}{V_t} = \frac{1}{2}$

0.463647 rounds down

$$T_{half} = \int_{0}^{V_t/2 \,\to\, 1/2} \frac{dV}{g(1+\underbrace{(V/V_t)^2}_{u^2})} = \boxed{\underbrace{\frac{V_t}{g}}_{K}\int_0^{1/2}\frac{du}{1+u^2}} \overset{b)}{=} \boxed{\arctan(1/2)\ \frac{V_t}{g}} \approx \boxed{0.4636\ \frac{V_t}{g}} \text{ yes!}$$

time units

dont grade c)

c) $\frac{vel}{acc} \sim \frac{length/\cancel{time}}{length/\cancel{time}^2} = \boxed{time!}$

arctan 0.99 !

d) $T_{99} = \int_0^{.99 V_t} \frac{dV}{g(1+(V/V_t)^2)} \overset{Maple}{=} \boxed{0.7804\ \frac{V_t}{g}}$

0.78037 rounds up