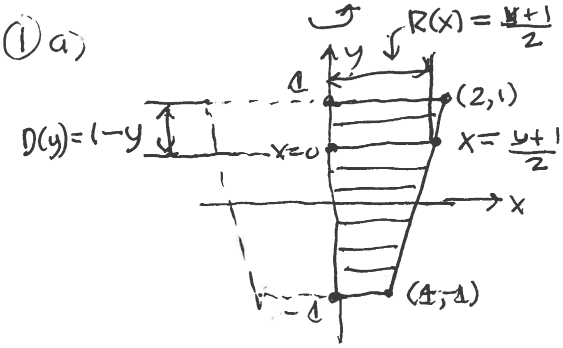
MAT1505-19F 03/04 Test 1 Answers

1 a)



$m = \frac{1-(-1)}{2-1} = 2$, pointslope:

$y - 1 = 2(x-2) = 2x-4 \to \boxed{y = 2x-3}$

$\to \boxed{x = \frac{y+3}{2}} = R(x)$

b) $A(x) = \pi R(x)^2 = \pi\left(\frac{3+y}{2}\right)^2 = \frac{\pi(y+3)^2}{4}$

$= \frac{\pi}{4}(y^2+6y+9)$

$V = \int_{-1}^{1} A(y)\,dy = \boxed{\int_{-1}^{1} \frac{\pi}{4}(y^2+6y+9)\,dy}$

$= \frac{\pi}{4}\left(\frac{y^3}{3} + \frac{6y^2}{2} + 9y\right)\Big|_{-1}^{1} = \frac{\pi}{4}\left(\frac{1}{3}+3+9 - \left(\frac{(-1)^3}{3} + 3(-1)^2 - 9\right)\right)$

$= \frac{\pi}{4}\left(\frac{2}{3} + 18\right) = \frac{\pi}{4}\left(\frac{56}{3}\right) = \boxed{\frac{14\pi}{3} \approx 14.6608}$

c) $W = \int_{-1}^{1} A(y)D(y)\,dy = \int_{-1}^{1} \frac{\pi}{4}(y^2+6y+9)(1-y)\,dy$

$= \boxed{\frac{11\pi}{3} \approx 11.5192}$

2 a) $y = te^{-kt} \to \frac{dy}{dt} = 1e^{-kt} + t(-k)e^{-kt}$

$= e^{-kt}(1-kt) = 0$

$\to t = 1/k \equiv t_p$

b) $M_{pp} = k\int_0^{1/k} t e^{-kt}\,dt = k\int_0^1 \left(\frac{T}{k}\right) e^{-T} \left(\frac{dT}{k}\right)$

$T = kt \to t = T/k$

$dT = k\,dt \to dt = dT/k$

$t = 0 \to T = 0$

$t = 1/k \to T = 1$

change of variable

$= \boxed{\frac{1}{k}\int_0^1 Te^{-T}\,dT} = \frac{1}{k}\Big[-Te^{-T} - \underbrace{\int -e^{-T}\,dT}_{-e^{-T}}\Big]\Big|_0^1$

c)

$u = T$, $dv = e^{-T}\,dt$

$du = dT$, $v = -e^{-T}$

2 c) continued

$M_{pp} = \frac{1}{k}\left[-Te^{-T} - e^{-T}\right]_0^1$

$= \left(\frac{1}{k}\right) - (1+T)e^{-T}\Big|_0^1$

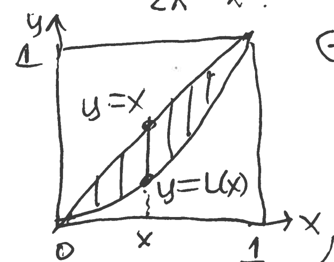
$= \frac{1}{k}\left(-2e^{-1} + e^0\right) = \boxed{\frac{1-2e^{-1}}{k}}$

d) $\boxed{\approx .2642\left(\frac{1}{k}\right)}$

3 a) $L(x) = x^2[1-(1-x)^2]$

$= x^2[1-(1-2x+x^2)] = x^2[2x - x^2]$

$= 2x^3 - x^4$



$G = 2\int_0^1 (x - L(x))\,dx$

$= 2\int_0^1 x - (2x^3 - x^4)\,dx$

$= 2\int_0^1 x - 2x^3 + x^4\,dx$

$= 2\left(\frac{x^2}{2} - \frac{2x^4}{4} + \frac{x^5}{5}\right)\Big|_0^1 = 2\left(\frac{1}{2} - \frac{1}{2} + \frac{1}{5}\right)$

$\boxed{= \frac{2}{5} = 0.4} \stackrel{?}{=} 0.4000$

b) $L_{avg} = \frac{1}{1/2}\int_{1/2}^{1} L(x)\,dx$

$= 2\int_{1/2}^{1} 2x^3 - x^4\,dx = 2\left(\frac{2}{4}x^4 - \frac{x^5}{5}\right)\Big|_{1/2}^{1}$

$= 2\left(\frac{1}{2} - \frac{1}{5} - \frac{1}{2}\left(\frac{1}{16}\right) + \frac{1}{5\cdot 32}\right) = 2\left(\underbrace{\frac{5-2}{10}}_{\frac{3}{10}} - \underbrace{\frac{1}{32}\left(1-\frac{1}{5}\right)}_{1/40}\right)$

$\frac{3}{10} - 1/40 = 11/40$

Maple $\boxed{\frac{11}{20} \approx 0.550}$

c) $2x^3 - x^4 = \frac{11}{20}$ numerically solve from $x = 1$!

$x = .7633092508$

$\approx \boxed{0.763}$