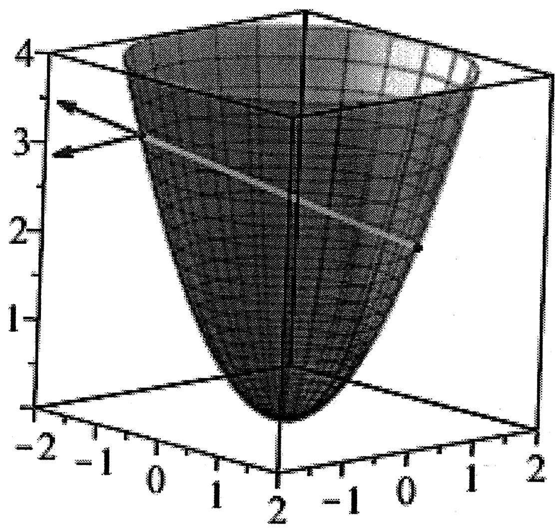
MAT2500-01/04 18S Quiz 6 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

a) Where does the normal line to the paraboloid $z=x^2+y^2$ at the point (1,1,2) intersect the paraboloid a second time?
b) At what angle does that normal line intersect the paraboloid at that second point?
[Hint: It is the complement of the acute angle between the normal line and the downward (=outward) normal at the point of intersection.]



▶ **solution**

a) $F(x,y,z) = z - x^2 - y^2 = 0$ level surface = graph of $f(x,y)=x^2+y^2$, $f(1,1)=2$

$\vec{r}_0 = \langle 1,1,2\rangle$

$\nabla F(x,y,z) = \langle -2x,-2y,1\rangle = \vec{n}(x,y,z)$

$\vec{n}(1,1,2) = \langle -2,-2,1\rangle$ $\hat{n}(1,1,2) = \frac{\langle -2,-2,1\rangle}{\sqrt{4+4+1}} = \frac{1}{3}\langle -2,-2,1\rangle$

options: can use $\pm\vec{n}, \pm\hat{n}$ as orientation vector

normal line upward from $\vec{r}_0$: $\vec{r}(t) = \vec{r}_0 + t\,\hat{n}(1,1,2) = \langle 1,1,2\rangle + \frac{t}{3}\langle -2,-2,1\rangle$

$= \langle 1-\frac{2}{3}t,\ 1-\frac{2}{3}t,\ 2+\frac{t}{3}\rangle = \langle x(t), y(t), z(t)\rangle$

$0 = F(x(t),y(t),z(t)) = 2+\frac{t}{3} - (1-\frac{2}{3}t)^2 - (1-\frac{2}{3}t)^2 = 2+\frac{t}{3} - 2(1-\frac{4t}{3}+\frac{4}{9}t^2) = \frac{t}{3}+\frac{8}{3}t-\frac{8}{9}t^2$

$= 3t - \frac{8}{9}t^2 = \frac{(27-8t)t}{9} \to t=0,\ t=\frac{27}{8}$

$\vec{r}_1 = \vec{r}(\frac{27}{8}) = \langle 1-\frac{2}{3}(\frac{27}{8}),\ 1-\frac{2}{3}(\frac{27}{8}),\ 2+\frac{1}{3}(\frac{27}{8})\rangle = \langle -\frac{10}{8}, -\frac{10}{8}, \frac{25}{8}\rangle = \frac{5}{8}\langle -2,-2,5\rangle$

$\boxed{=\langle -5/4, -5/4, 25/8\rangle}$

$=\langle -1.25, -1.25, 3.125\rangle$

b) $\vec{n}(\vec{r}_1) = \langle \frac{5}{2}, \frac{5}{2}, 1\rangle$ $\hat{n}(\vec{r}_1) = \frac{\langle 5,5,2\rangle}{\sqrt{54}} = \frac{\langle 5,5,2\rangle}{3\sqrt{6}}$

$\cos\phi = |\hat{n}(r_1)\cdot\hat{n}(1,1,2)| = \frac{1}{3\sqrt{6}}\left|\langle 5,5,2\rangle\cdot\frac{\langle -2,-2,1\rangle}{3}\right| = \frac{1}{9\sqrt{6}}|(-10-10+2)| = \left|\frac{18}{9\sqrt{6}}\right| = \left|\frac{2}{\sqrt{6}}\right| = \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$

want smaller, acute angle.

$\theta = \boxed{\frac{\pi}{2} - \arccos\left(\frac{2}{\sqrt{6}}\right)} \approx \boxed{54.7°}$

$= \arctan\frac{1}{\sqrt{2}}$! even simpler.

$\frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$ so $\phi = \arccos\sqrt{\frac{2}{3}} = \arctan\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{2} - \phi = \arccos\frac{1}{\sqrt{3}}$