MAT2500-01/04 18S Quiz 0. Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Consider the two spheres: $x^2+y^2+z^2=100$, $x^2+y^2+z^2=6\,x+8\,y+24\,z$.

a) What is the distance between the centers of these two spheres?

b) If $P_0(x_0, y_0, z_0)$ is the center of the second sphere, then the point P_1 with coordinates $(x_1, y_1, z_1)=(t\,x_0, t\,y_0, t\,z_0)$ with $t>0$ lies on the line segment connecting the two centers. What value of t corresponds to the point on the first sphere and what are the coordinates of this point? This is the point on the first sphere closest to the center of the second sphere.

c) What fraction of the distance between the centers does the separation of the two points P_0, P_1 represent?

d) What is the highest point on the larger sphere?

e) Make a rough hand sketch of a plane cross-section through the line joining the centers (made horizontal for the 2d diagram) and locate the two circles in that plane with their radii and separation. Does your answer to part c) look reasonable?

▶ solution

Clever observation. An obvious soln of the second eqn is $(x,y,z)=(0,0,0)$ because there is no constant term, which means the second sphere passes through to origin which is the center of the first sphere so their separation is the second sphere radius and the difference of their radii is the separation distance between one center and the other sphere.

a) $x^2-6x+y^2-8y+z^2-24z=0$

$(x-3)^2-3^2+(y-4)^2-4^2+(z-12)^2-12^2=0$

$(x-3)^2+(y-4)^2+(z-12)^2=\underbrace{\underbrace{3^2+4^2}_{5^2}+12^2}_{13^2}=169=13^2$

center: $(3,4,12)=P_0$

radius: 13

$x^2+y^2+z^2=10^2$ center $(0,0,0)$ radius: 10

distance between centers: $|\overrightarrow{OP_0}|=\sqrt{3^2+4^2+12^2}=\sqrt{169}=\boxed{13}$

b) (x_1,y_1,z_1) must satisfy eqn of first sphere: $(tx_0)^2+(ty_0)^2+(tz_0)^2=100$

$(3t)^2+(4t)^2+(12t)^2=100$

$(3^2+4^2+12^2)t^2=100\rightarrow$ $13^2t^2=10^2$

$13t=10\rightarrow\boxed{t=\frac{10}{13}}$

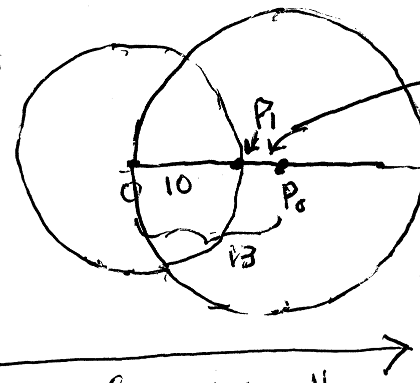
$(x_1,y_1,z_1)=\left(\frac{10}{13}3,\frac{10}{13}4,\frac{10}{13}12\right)=\boxed{\left(\frac{30}{13},\frac{40}{13},\frac{120}{13}\right)}$

c) $t=\frac{10}{13}$ is the fraction of the distance from O to P_0 (note $t=1$ corresponds to P_0) so $1-t=1-\frac{10}{13}=\boxed{\frac{3}{13}}$ is the remaining fraction

d) Add radius to z-coordinate of center to get to highest pt

N: $(3,4,12+13)=\boxed{(3,4,25)}$

e)



$13-10=3$

$\downarrow$

fraction of total: $\frac{3}{13}$

confirms c) result

So must be reasonable!