MAT2705-04/05 18F Quiz 6 Print Name (Last, First) ____________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. Let $\vec{b}_1 = \langle -1, 2\rangle$, $\vec{b}_2 = \langle -3, -2\rangle$ be a new basis of the plane, with basis changing matrix $B = \langle \vec{b}_1 | \vec{b}_2 \rangle$.

a) On the grid provided draw in the arrows at the origin representing these two basis vectors, then extend them to new coordinate axes and label them y_1, y_2 with arrowheads at the positive end. Mark tickmark multiples along each.

b) Draw in the position vector $\vec{a} = \langle 7, 10\rangle$ and line segments parallel to the new axes which reach back to those axes from its tip to create a parallelogram for which it is the main diagonal and use this parallelogram to read off the multiples of the two basis vectors which sum to $\vec{a}$, i.e., its new coordinates $\langle y_1, y_2\rangle$. Mark the vector sides of this parallelogram along the coordinate axes with these multiples $y_1\vec{b}_1$ and $y_2\vec{b}_2$.

c) Write down in matrix form explicitly the linear system $B\vec{y} = \vec{a}$. State the inverse matrix and use it to solve this system for the unknown coordinates $\langle y_1, y_2\rangle$.

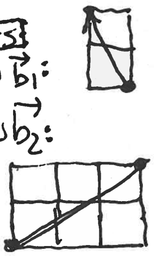
d) Draw in an arrow representing the position vector $\vec{c}$ whose new coordinates are instead $\langle y_1, y_2\rangle = \langle 3, 2\rangle$ (and label it as such), together with its corresponding parallelogram back to the new coordinate axes. Read off its Cartesian coordinates from the grid.

e) Use matrix multiplication to obtain the old coordinates from these new coordinates.

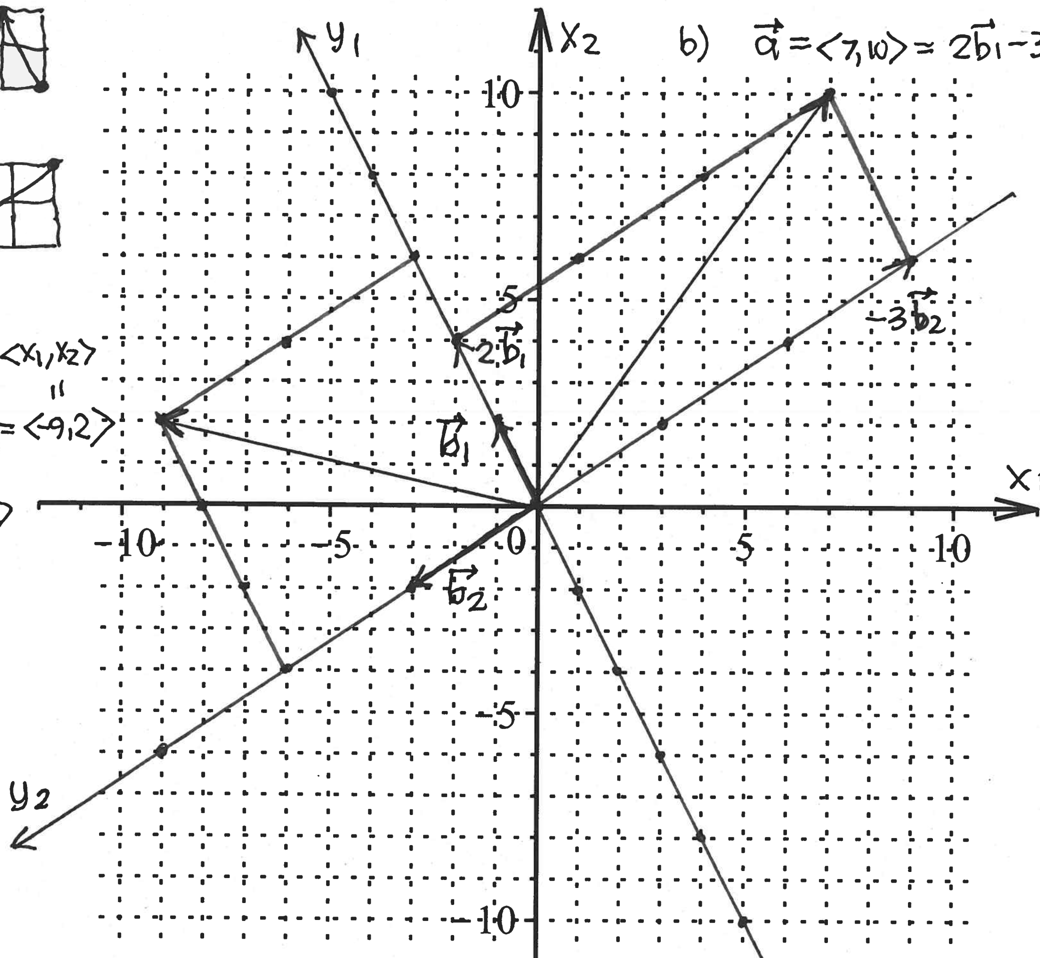
$\boxed{\text{movements:}}$

parallel to $\vec{b}_1$:

parallel to $\vec{b}_2$:



b) $\vec{a} = \langle 7, 10\rangle = 2\vec{b}_1 - 3\vec{b}_2 \to \langle y_1, y_2\rangle = \langle 2, -3\rangle$



d) $\langle x_1, x_2\rangle$

$\vec{c} = 3\vec{b}_1 + 2\vec{b}_2 = \langle -9, 2\rangle$

$\langle y_1, y_2\rangle = \langle 3, 2\rangle$

c) $B = \langle \vec{b}_1 | \vec{b}_2\rangle = \begin{bmatrix} -1 & -3 \\ 2 & -2 \end{bmatrix}$ $\quad B^{-1} = \frac{1}{2-(-6)}\begin{bmatrix} -2 & 3 \\ -2 & -1 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 2 & -3 \\ 2 & 1 \end{bmatrix}$

$y_1\begin{bmatrix} -1 \\ 2 \end{bmatrix} + y_2\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & -3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \end{bmatrix}} \to \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 2 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 10 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} 14-30 \\ 14+10 \end{bmatrix} = -\frac{1}{8}\begin{bmatrix} -16 \\ 24 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}$ agrees with b)

e) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3-6 \\ 6-4 \end{bmatrix} = \boxed{\begin{bmatrix} -9 \\ 2 \end{bmatrix}}$ agrees with d).