

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations if appropriate for interpretation). Indicate where technology is used and what type (Maple, GC). Always justify claims.

1. $\frac{dy}{dx} = -2y + 3e^{-3x}$, $y(0) = 1$, $x \geq 0$.

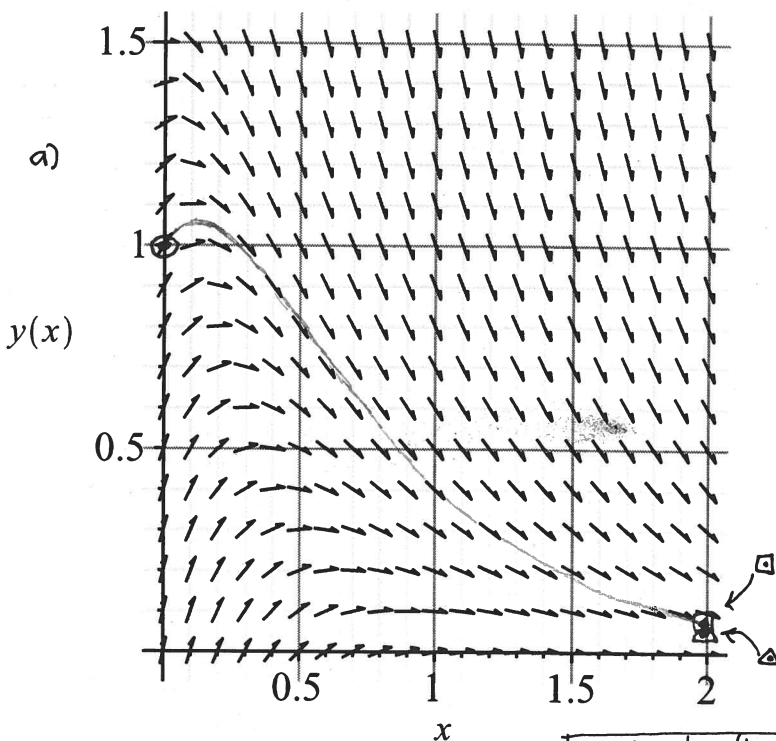
a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition. Put a squared dot on the curve at $x = 2$. Estimate your approximate value of $y(2)$.

b) Use the linear solution recipe to find the general solution of this differential equation. Simplify it and box it.

c) Find the solution of this differential equation which satisfies the given initial condition. Simplify it and box it.

d) Evaluate $y(2)$ numerically to 3 decimal places and mark the corresponding point on your graph with a triangled dot. Is this consistent with your part a) result? Explain.

e) Does your initial value problem solution agree with Maple (which requires Expand to multiply out, and Simplify, Simplify to simplify)? If not, can you find your mistake? If so, show the equivalence of your solution with Maple's. Did you simplify your solution as requested before comparing it to Maple's?



graphical estimate $y(2) \approx 0.8$

notice that the grid spacing is 0.1

don't omit this step!

① b) $e^{2x} \left[\frac{dy}{dx} + 2y = 3e^{-3x} \right] \rightarrow \frac{d}{dx} (e^{2x} y) = 3e^{2x} e^{-3x} = 3e^{-x}$

$\int 2 dx = 2x$
 e^{2x}

$e^{2x} y = \int 3e^{-x} dx = \frac{3e^{-x}}{-1} + C = -3e^{-x} + C$

$y = e^{-2x} (C - 3e^{-x}) = Ce^{-2x} - 3e^{-3x}$ gen soln.

c) $y(0) = 1 \Leftrightarrow x=0, y=1 \quad 1 = Ce^0 - 3e^0 = C - 3 \rightarrow C = 4 \rightarrow y = 4e^{-2x} - 3e^{-3x}$ IVP soln

d) $y(2) = 4e^{-4} - 3e^{-6} \approx 0.06586 \approx 0.066$

my graphical estimate seems reasonably close considering the grid box size & the viewing window.

NOTE: $e^{-2x} (C - 3e^{-x})$ is NOT simplified compared to $Ce^{-2x} - 3e^{-2x}e^{-x} = Ce^{-2x} - 3e^{-3x}$ which is trivial to either integrate or differentiate.

The product expression must be multiplied out & combined to integrate and it would be a poor choice to differentiate it with the product rule instead of multiplying out first. Similarly $\frac{1}{e^{2x}}$ must be "simplified" to e^{-2x} . The parenthetical remark explains what the simplified expression should be.

In the future $\int (\dots)$ instead of $\int (\dots) dx$ (matching opening & closing "delimiters" for the integrand) will result in a loss of points.