MAT2705-04/05 18F Quiz 2 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Always justify your claims.

1. $\frac{dy}{dx} = (y-1)\sin(x), y\left(-\frac{3}{2}\right) = -2.$

Note the 0.2 separation in the arrow grid.

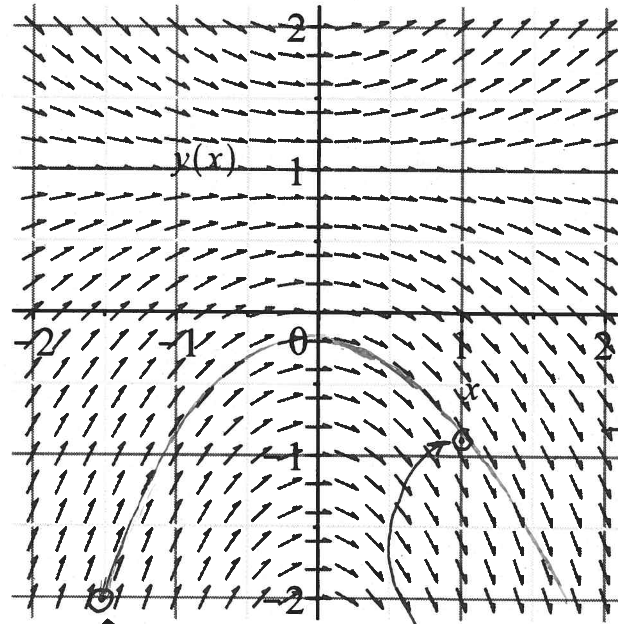
a) Indicate the initial data point on the graph by a circled dot annotated by an arrow pointing to the point from the initial condition written to the side of the graph and roughly draw in the corresponding solution curve.

b) Find the general solution of this separable differential equation.

c) What solution is obviously missing from your family (see direction field)?

d) Find the solution which satisfies the initial condition. ~~How does this compare visually to your hand drawn curve?~~

e) Evaluate $y(1)$ for this solution and mark the corresponding point on the graph by a circled dot annotated as above (arrow from $y(1) \approx$...). Is this consistent with your approximate hand drawn solution? Explain (estimate your value from the curve, compare).



$y=-0.$

$(-3/2,-2)$ $y(-3/2)=-2$ $y(1) \approx -0.875$

f) Check by hand that your solution to d) solves the differential equation. [Remember, backsub everywhere in the DE eliminating y, then simplify both sides independently.]

g) Enter the differential equation (with the right hand side factored or trouble results!) and the initial condition separated by a comma in Maple. Use the right side menu to solve it. Write down exactly the form of the solution that it gives you. Does it agree with your hand solution? Explain why if so. If not, you better find your error.

▶ solution

(1) b) $\frac{dy}{dx} = (y-1)\sin x$ separable

$\int \frac{dy}{y-1} = \int \sin x \, dx$

$\ln|y-1| = -\cos x + C_1$

$e^{\ln|y-1|} \quad e^{-\cos x + C_1}$

$|y-1| = e^{C_1 - \cos x} = e^{C_1} e^{-\cos x}$

$y-1 = \underbrace{\pm e^{C_1}}_{C} e^{-\cos x}$

$\boxed{y = 1 + C e^{-\cos x}}$ gen soln

c) $C=0$ is not allowed by soln technique because of division by $y-1$ so $y \neq 1$. By redefinition of C_1 to C we recapture this soln $y=1$. Obvious as an isocline solution in the graph.

d) $-2 = y(-3/2) = 1 + C e^{-\cos(-3/2)} = 1 + C e^{-\cos 3/2}$

$C = (-2-1) e^{\cos 3/2} = -3 e^{\cos 3/2}$

$\boxed{y = 1 - 3e^{\cos 3/2} e^{-\cos x} = 1 - 3e^{\cos 3/2 - \cos x}}$ would have to plot to compare. no time

e) $y(1) = 1 - 3e^{\cos 3/2 - \cos 1} \approx \boxed{-0.8758}$

my soln curve crosses between this value of y and $y=-0.8$ so pretty close.

f) $\frac{d}{dx}\left(y = 1 - 3e^{\cos 3/2 - \cos x}\right)$

$\frac{dy}{dx} = -3e^{\cos 3/2 - \cos x} (\sin x)$

$-3\sin x \, e^{\cos 3/2 - \cos x} = (+1 - 3e^{\cos 3/2 - \cos x} - 1)\sin x$

$= -3e^{\cos 3/2 - \cos x}$ ✓

g) $\boxed{y = 1 - \frac{3e^{-\cos x}}{e^{-\cos 3/2}}}$ agrees when combine exponentials.