MAT2705-04/05 18f Quiz 1 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\dfrac{dy}{dx} = 3x^2(y^2+1)$, *gen soln* : $y = \tan(x^3 + C)$

a) Verify that this y satisfies the given differential equation. [Hint: recall that $\tan' = \sec^2$, $\tan^2 + 1 = \sec^2$.]

b) Find the solution which satisfies the initial condition $y(0) = 1$.

Organize your work as though you were playing professor.

Note: Math is "case sensitive" upper and lower case letters are not interchangeable: $A \neq a$. Be sure to follow established conventions.

2. a) Choose ***appropriately*** named variables and write a differential equation that models the situation:

"The acceleration of a Lamborghini is proportional to the difference between 250 km/h and the velocity of the car."

b) What sign should your constant of proportionality have?

c) OPTIONAL. Does this DE make sense for $v > 250$ or $v < 0$? Explain.

▶ solution

replace y & $\frac{dy}{dx}$ by their expressions everywhere in DE, then simplify both sides

① a) $\frac{d}{dx}\left[y = \tan(x^3+C)\right]$

$\frac{dy}{dx} = \sec^2(x^3+C)(3x^2+0)$

$= 3x^2\sec^2(x^3+C)$

$y^2+1 = \tan^2(x^3+C)+1$

$= \sec^2(x^3+C)$

$\frac{dy}{dx} = 3x^2(y^2+1) \rightarrow 3x^2\sec^2(x^3+C) = 3x^2(\sec^2(x^3+C))$ ✓

OR

$\frac{dy}{dx} = 3x^2(y^2+1) \rightarrow 3x^2\sec^2(x^3+C) = 3x^2(\tan^2(x^3+C)+1)$

$\downarrow$

$3x^2\sec^2(x^3+C) = 3x^2(\sec^2(x^3+C))$ ✓

Do not "manipulate" the eqn by dividing, multiplying, adding or subtracting to each side — ONLY "simplify" using rules of algebra, identities for special functions.

b) $y(0)=1 \leftrightarrow x=0, y=1 \longrightarrow y = \tan(x^3+C)$

$1 = \tan(0+C) = \tan C$

not soln of DE $\not\rightarrow$ $C = \arctan 1 = \pi/4$

$\boxed{y = \tan(x^3 + \pi/4)}$

Box it.

goal is to obtain a final expression for unknown y.

② a) appropriately named variables: t for time, v for velocity

acceleration $a = \frac{dv}{dt} \propto 250 - v \longrightarrow \boxed{\frac{dv}{dt} = k(250-v)}$

a D.E. that can be solved can only have one unknown in it. $a = k(250-v)$ cannot be solved without knowing how a & v are related.

b) $\frac{dv}{dt} = k(250-v)$

≥ 0 as increasing speed from 0

want $\frac{dv}{dt} > 0$ so $\boxed{k>0}$

If you write $\frac{dv}{dt} = k_2(v-250)$ then $k_2<0$ BUT better to have an explicit negative sign $k_2 = -k$

$\frac{dv}{dt} = -k(v-250) = k(250-v)$

simplest

c) If $v > 250$ then the DE says $\frac{dv}{dt} < 0$ so the car decelerates, which doesn't make sense. A car cannot go faster than its top end speed

If $v < 0$, then $k(250-v) > 0$ so the car should accelerate in the opposite direction? I dont think so.

Clearly the wording of the defining statement is intended to refer instead to the speed $|v| \geq 0$ independent of directionality of the 1D motion. This is where "common sense" enters the problem. So the D.E. is only valid for $0 \leq v \leq 250$.