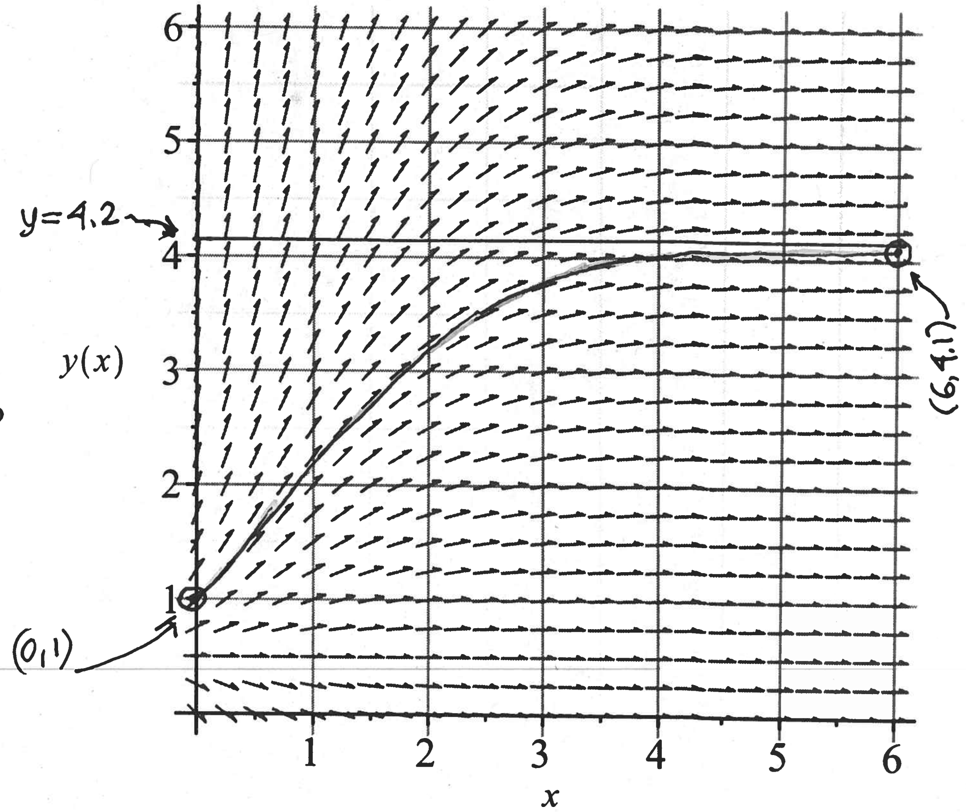
MAT2705-04/05 18F Test 1 Print Name (Last, First) ______________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). [Recall you need $y'(t), y(t)$ instead of y', y in your differential equation for an unknown variable y for Maple to interpret the prime as a t derivative.] Always justify claims. ***Start each problem by writing down the Maple solution of the initial value problem.***

1. $e^x \frac{dy}{dx} = 2y - 1, y(0)=1$

a) Locate the initial data point on the graph with a circled dot and label it with its coordinates.
b) Sketch the solution curve through this point and estimate the value of y at $x = 6$, labeling that point as in part a).
c) Use the linear first order DE solution algorithm to find the general solution.**
d) Find the solution of the initial value problem.
e) Verify that indeed this IVP solution is a solution of the DE by backsubstitution into the original form of the DE (replace the unknown everywhere in the equation and simplify both sides independently).
f) Evaluate this solution exactly and approximately at $x = 6$. How does your estimate in part a) compare with this value?



g) Evaluate the limit y_∞ as $x \rightarrow \infty$ of this solution exactly and approximately to 3 significant figures. Draw in the line $y = y_\infty$ on the graph. What is the significance of this line?

** A simple u-substitution will enable you to do the required final integration, but if you have trouble doing that, use a Maple antiderivative.

2. $\frac{dQ}{dt} - 2\sin(t)\, e^{-Q} = 0, \quad Q(0) = 0$

a) Find the general solution of this differential equation.
b) Find the solution of the initial value problem.
c) Plot your solution for $t = 0..2\pi$ and make a rough hand sketch of the result, labeling axes and tickmarks.
d) Where does the first maximum occur for $t > 0$? What are the exact coordinates of this point? What is the maximum value to 5 significant digits?

pledge **[Be sure to sign and date the pledge before handing in this test.]**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

MAT2705-04/05 18F Test 1 Answers

① c) $e^x \frac{dy}{dx} = 2y-1$

$$\frac{dy}{dx} = e^{-x}(2y-1) = 2e^{-x}y - e^{-x}$$

$$\left[\frac{dy}{dx} \underbrace{-2e^{-x}}\, y = -e^{-x}\right]$$

$$\underbrace{\int -2e^{-x}\,dx} = \frac{-2e^{-x}}{-1} = 2e^{-x}$$

exp $\longrightarrow$ $= e^{2e^{-x}}$

$$\frac{d}{dx}\left(ye^{2e^{-x}}\right) = -e^{-x}e^{2e^{-x}}$$

$$ye^{2e^{-x}} = \int -e^{\underset{\swarrow}{2e^{-x}}} e^{-x}\,dx + C$$

$$\left(\begin{array}{l} u = 2e^{-x} \\ du = -2e^{-x}\,dx \\ \frac{1}{2}du = -e^{-x}\,dx \end{array}\right)$$

$$= \frac{1}{2}\int e^u\,du + C = \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{2e^{-x}} + C$$

$$\boxed{\begin{aligned} y &= e^{-2e^{-x}}\left(\tfrac{1}{2}e^{2e^{-x}} + C\right) \\ &= \tfrac{1}{2} + Ce^{-2e^{-x}} \end{aligned}}$$ gen soln.

d) $1 = y(0) = \frac{1}{2} + Ce^{-2}$

$\frac{1}{2} = Ce^{-2}$, $C = \frac{1}{2}e^2$

$$\boxed{\begin{aligned} y &= \tfrac{1}{2} + \tfrac{1}{2}e^2 e^{-2e^{-x}} \\ &= \tfrac{1}{2}\left(1 + e^{2(1-e^{-x})}\right) \end{aligned}}$$ IVP soln

e) $\frac{dy}{dx} = 0 + \frac{1}{2}e^2 e^{-2e^{-x}}\left(-2e^{-x}(-1)\right)$

$= e^2 e^{-x} e^{-2e^{-x}}$

$$\cancel{e^x}\left(e^2\cancel{e^{-x}}_{1} e^{-2e^{-x}}\right) = 2\left(\tfrac{1}{2} + \tfrac{1}{2}e^2 e^{-2e^{-x}}\right) - 1$$

$$e^2 e^{-2e^{-x}} = 1 + e^2 e^{-2e^{-x}} - 1$$

$$= e^2 e^{-2e^{-x}} \quad ✓$$

f) $y(6) = \frac{1}{2} + \frac{1}{2}e^2 e^{-2e^{-6}} \approx 4.17626 \approx 4.18$

pretty close to 4.1 estimate from graph
(grid divisions are 0.25)

① g) $\lim\limits_{t\to\infty} y = \lim\limits_{t\to\infty} \frac{1}{2}\left(1 + e^{2(1-\overset{\searrow}{e^{-x}}_{0})}\right) = \boxed{\frac{1}{2}(1+e^2)} \approx 4.19452$

$y_\infty =$ $\approx \boxed{4.19}$

The line $y = y_\infty$ is a horizontal asymptote for the soln curve.

② a) $\frac{dQ}{dt} - 2\sin t\, e^{-Q} = 0 \rightarrow \frac{dQ}{dt} = 2\sin t\, e^{-Q}$ separable

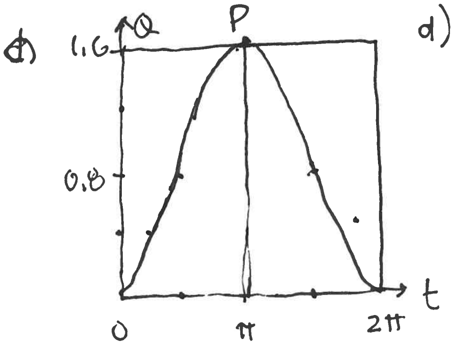
$\int e^Q\,dQ = \int 2\sin t\,dt$ separate & integrate

$e^Q = -2\cos t + C$, $\boxed{Q = \ln(C - 2\cos t)}$ gen soln

b) $0 = Q(0) = \ln(C - 2\cos 0) = \ln(C-2)$

$1 = e^0 = e^{\ln(C-2)} = C - 2 \rightarrow C = 3$, $\boxed{Q = \ln(3 - 2\cos t)}$

d)



P has coords: $\boxed{(\pi, \ln 5) \approx (3.1416, 1.6094)}$

d) $\frac{dQ}{dt} = \frac{1}{3 - 2\cos t}(0 + 2\sin t) = 0$

$\sin t = 0$, $t = 0, \pi, 2\pi$

max obviously at $t = \pi$

$Q(\pi) = \ln(3 - 2\cos\pi)$

$= \ln(3+2) = \boxed{\ln 5}$

≈ 1.60943

$\approx \boxed{1.6094}$

5 sig figs!