MAT2500-01/04 17S Quiz 6 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Show that $f(x, t) = \sin(x - 2t)$ satisfies the wave equation $f_{xx} = \frac{1}{4} f_{tt}$.

2. The length and width of a rectangle are measured as 30 and 24 cm with an error in measurement of at most 0.1 cm.

a) Use differentials to estimate the maximum error in the ratio of its length to its width. As always, introduce obvious symbols for your variables and state their values and their differentials used to do this estimation. State the general differential and and use it to do the estimate (remember the triangle inequality and absolute values of the differentials).

b) What are the error bars correct to 5 decimal places that constrain the computed ratio values: $R_1 \le R \le R_2$

[The error bar interval has endpoints obtained by adding and subtracting the maximum error from the calculated value.]

c) **Optional.** Evaluate the numerical error bars for the actual maximum and minimum computed values of the ratio to 5 decimal places to compare with your estimated values.

▶ solution

① $f(x,t) = \sin(x-2t)$

$f_x(x,t) = \frac{\partial}{\partial x}\sin(x-2t) = \cos(x-2t)\underbrace{\frac{\partial}{\partial x}(x-2t)}_{1}$
$= \cos(x-2t)$

$f_t(x,t) = \frac{\partial}{\partial t}\sin(x-2t) = \cos(x-2t)\underbrace{\frac{\partial}{\partial t}(x-2t)}_{-2}$
$= -2\cos(x-2t)$

$f_{xx}(x,t) = \frac{\partial}{\partial x}\cos(x-2t) = -\sin(x-2t)\underbrace{\frac{\partial}{\partial x}(x-2t)}_{1}$
$= -\sin(x-2t)$

$f_{tt}(x,t) = \frac{\partial}{\partial t}(-2\cos(x-2t)) = -2(-\sin(x-2t))\underbrace{\frac{\partial}{\partial t}(x-2t)}_{-2}$
$= -4\sin(x-2t)$

$\frac{1}{4}f_{tt}(x,t) = \frac{1}{4}(-4\sin(x-2t)) = -\sin(x-2t)$
$= f_{xx}(x,t)$ ✓

② a) $\ell = 30 \quad |d\ell| \le 0.1$
$w = 24 \quad |dw| \le 0.1$

$R = \frac{\ell}{w} \qquad R\Big|_{\substack{\ell=30\\w=24}} = \frac{30}{24} = \frac{5}{4} = 1.25$

$\frac{\partial R}{\partial \ell} = \frac{\partial}{\partial \ell}\left(\frac{\ell}{w}\right) = \frac{1}{w}, \quad \frac{\partial R}{\partial w} = \frac{\partial}{\partial w}(\ell w^{-1}) = -\ell w^{-2}$

$\boxed{dR = \frac{\partial R}{\partial \ell}d\ell + \frac{\partial R}{\partial w}dw = \boxed{\frac{1}{w}d\ell - \frac{\ell}{w^2}dw}}$ general differential

$|dR| = \left|\frac{d\ell}{w} - \frac{\ell}{w^2}dw\right| \le \frac{|d\ell|}{w} + \frac{\ell|dw|}{w^2}$

$|dR|_{\substack{\ell=30\\w=24\\|d\ell|=0.1\\|dw|=0.1}} \le \frac{0.1}{24} + \frac{30}{24^2}0.1 = \frac{0.1}{24}(1+1.25) = 0.009375 \approx \boxed{0.00938}$ 5 decimals

$R \pm |dR|_{*}$: $1.25000 \pm 0.00938 = 1.24062,\ 1.25938$

so $\boxed{1.24062 \le R \le 1.25938}$ is the linear estimate of the "error bar" around 1.25

c) $R_{min} = \frac{29.9}{24.1} \approx 1.24066$

so $\boxed{1.24066 \le R \le 1.25941}$ "exact" error bar

$R_{max} = \frac{30.1}{23.9} \approx 1.25941$