MAT2500-01/04 17s Quiz 4 Take Home Print Name (Last, First) ________________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC).

1. a) Find the linear approximation to the function $f(x,y,z) = x^3\sqrt{y^2+z^2}$ at the point (2,3,4) and

b) use it to approximate $Q = (1.98)^3\sqrt{(3.01)^2+(3.97)^2}$ to 3 decimal places and compare this approximate value to the numerical value of $Q$ to 3 decimal places. What is the percentage error in this approximation?

c) Evaluate $f_{yz}(x,y,z)$ and $f_{yz}(2,3,4)$ .

2. For the ellipsoid $36x^2+9y^2+4z^2=49$, use implicit differentiation to evaluate $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ as functions of $(x,y,z)$ and their values at the point (1,1,1).

## ▶ solution

1 a) $f(x,y,z) = x^3(y^2+z^2)^{1/2}$ $\quad f(2,3,4) = 2^3(3^2+4^2)^{1/2} = 8\cdot 5 = 40$

$f_x(x,y,z) = \frac{\partial}{\partial x}\left(x^3(y^2+z^2)^{1/2}\right) = 3x^2(y^2+z^2)^{1/2}$ $\quad f_x(2,3,4) = 3\cdot 4\cdot 5 = 60$

$f_y(x,y,z) = \frac{\partial}{\partial y}\left(x^3(y^2+z^2)^{1/2}\right) = x^3\left(\frac{1}{2}\right)(y^2+z^2)^{-1/2}(2y+0) = \frac{x^3y}{(y^2+z^2)^{1/2}}$ $\quad f_y(2,3,4) = \frac{8\cdot 3}{5} = \frac{24}{5}$

$f_z(x,y,z) = \frac{\partial}{\partial z}\left(x^3(y^2+z^2)^{1/2}\right) = x^3\left(\frac{1}{2}\right)(y^2+z^2)^{-1/2}(0+2z) = \frac{x^3z}{(y^2+z^2)^{1/2}}$ $\quad f_z(2,3,4) = \frac{8\cdot 4}{5} = \frac{32}{5}$

$\boxed{L(x,y,z)} = f(2,3,4) + f_x(2,3,4)(x-2) + f_y(2,3,4)(y-3) + f_z(2,3,4)(z-4)$

$= \boxed{40 + 60(x-2) + \frac{24}{5}(y-3) + \frac{32}{5}(z-4)}$

b) $Q \approx L(1.98, 3.01, 3.97) = 40 + 60(1.98-2.00) + \frac{24}{5}(3.01-3.00) + \frac{32}{5}(3.97-4.00)$

$= 40 + 60(-0.02) + \frac{24}{5}(0.01) + \frac{32}{5}(-0.03) = 40 + .01\left(-60(2) + \frac{24}{5} - \frac{32(3)}{5}\right)$

$\approx 40 - 1.344 = \boxed{38.656}$

$\downarrow$

$Q = f(1.98, 3.01, 3.97) \approx \boxed{38.673}$ differ in 2nd decimal place, linear approx. too high

$\frac{L(1.96, 3.01, 3.97) - f(1.98, 3.01, 3.97)}{f(1.98, 3.01, 3.97)} \approx \frac{-.017}{38.673} \approx -0.00044$ so about $\boxed{-.04\%}$, very small

c) $f_{yz}(x,y,z) = \frac{\partial}{\partial z}\left(x^3y(y^2+z^2)^{-1/2}\right) = x^3y\left(-\frac{1}{2}\right)(y^2+z^2)^{-3/2}(0+2z) = \boxed{-\frac{x^3yz}{(y^2+z^2)^{3/2}}}$

$f_{yz}(2,3,4) = -\frac{8(3)(4)}{5^3} = \boxed{-\frac{96}{125}}$

2 $\frac{\partial}{\partial x}\left[36x^2+9y^2+4z^2 = 49\right] \to 36(2x) + 4\left(2z\frac{\partial z}{\partial x}\right) = 0 \to \frac{\partial z}{\partial x} = -\frac{36(2x)}{4(2z)} = \boxed{-\frac{9x}{z}}$

$\frac{\partial}{\partial y}\left[\quad \downarrow \quad\right] \to 9(2y) + 4\left(2z\frac{\partial z}{\partial y}\right) = 0 \to \frac{\partial z}{\partial y} = \frac{-9(2y)}{4(2z)} = \boxed{-\frac{9}{4}\frac{y}{z}}$

$\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)} = \boxed{-9}$ $\qquad \left.\frac{\partial z}{\partial y}\right|_{(1,1,1)} = \boxed{-\frac{9}{4}}$