MAT2500-01/04 17s Quiz 4 Take Home Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC).

1. The temperature-humidity index I (or humidex, for short) is the perceived air temperature when the actual temperature is T and the relative humidity is h, so we can write $I = f(T, h)$. The following table of values of I is an excerpt from a table compiled by the National Oceanic & Atmospheric Administration.

(a) What is the value of $f(95, 20)$? What is its meaning?
(b) For what value of h is $f(80, h) = 82$? Formulate this question in words.
(c) For what value of T is $f(T, 40) = 86$? Formulate this question in words.
(d) What are the meanings of the functions $I = f(85, h)$ and $I = f(100, h)$? Compare the behavior of these two functions of h, *i.e.*, as h increases, how does I behave for the second case compared to the first case? [Hint: compare the successive differences of the two functions.]

Apparent temperature as a function of temperature and humidity

Actual temperature (°F) \ Relative humidity (%)	20	30	40	50	60	70
80	77	78	79	81	(82)	83
85	82	84	(86)	88	90	93
90	87	90	93	96	100	106
95	(93)	96	101	107	114	124
100	99	104	110	120	132	144

(e) Evaluate the average rate of change for $f(100, h)$ for the intervals $h = 30..40$ and then $h = 40..50$ and then average these to get a decimal value for the "instantaneous" rate of change of $f(100, h)$ at $h = 40$:

$$\frac{d}{dh} f(100, h) \Big|_{h=40}.$$

f) Using this result, by how much would you expect the perceived temperature of 110 degrees F to increase if the humidity increases from 40% to 42% at an actual temperature of 100 degrees F?

▶ solution

1a) $f(95,20) = 93$. When the actual temperature is 95°F and the humidity is 20%, the perceived temperature is 93°F.

b) $f(80,60) = 82$. When the actual temperature is 80°F, at what humidity is the perceived temperature 82°F?
$\boxed{h = 60}$

c) $f(85,40) = 86$ When the humidity is 40%, at what actual temperature is the perceived temperature 86°F?
$\boxed{T = 85}$

d) $f(85,h)$ is the perceived temperature at 85°F (actual temperature) as a function of the humidity.

$f(100,h)$ is instead the perceived temperature at 100°F as a function of the humidity.

Note increments for $f(85,h)$: $\Delta I \to$ 2, 2, 2, 2, 3 — $f(85,h)$ is increasing steadily but
for $f(100,h)$: $\Delta I \to$ 5, 6, 10, 12, 12 — $f(100,h)$ is increasing faster and faster

e)

h	30	40	50
I	104	110	120

$\Delta h = 10$; $\Delta I_- = 6$; $\Delta I_+ = 10$

$\frac{\Delta I_-}{\Delta h} = \frac{6}{10} = .6$ $\quad \frac{\Delta I_+}{\Delta h} = \frac{10}{10} = 1.0$

$\left(\frac{\Delta I}{\Delta h}\right)_{avg} = \frac{.6 + 1.0}{2} = \frac{1.6}{2} = \boxed{0.8}$ °F/% pt

f) $\Delta I \approx \left(\frac{\Delta I}{\Delta h}\right)_{avg} \Delta h = (0.8)(2) = \boxed{1.6\,°F}$ (Thus increasing to 111.6°F.)