MAT2500-01/04 17S Test 2 Answers

① a) $f(x,y) = xye^{-x-y}$ $\quad P(2,-2)$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}(xye^{-x-y}) = ye^{-x-y} + xy\,e^{-x-y}(-1)$$
$$= ye^{-x-y}(1-x)$$
$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(xye^{-x-y}) = xe^{-x-y} + xye^{-x-y}(-1)$$
$$= xe^{-x-y}(1-y)$$

$$\vec{\nabla} f(x,y) = \left\langle \frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y) \right\rangle$$
$$= \boxed{\langle y(1-x)e^{-x-y}, x(1-y)e^{-x-y} \rangle}$$
$$\vec{\nabla} f(2,-2) = \langle -2(1-2)e^{-2+2}, 2(1+2)e^{-2+2} \rangle$$
$$= \boxed{\langle 2, 6 \rangle} = 2\langle 1,3 \rangle$$

b) $$\widehat{\vec{\nabla} f(2,-2)} = \frac{\langle 1,3 \rangle}{\sqrt{1+3^2}} = \boxed{\frac{\langle 1,3 \rangle}{\sqrt{10}}} \equiv \hat{u}$$

$$|\vec{\nabla} f(2,-2)| = 2\sqrt{1+3^2} = \boxed{2\sqrt{10}} = D_{\hat{u}} f(2,-2)$$

c) $$\vec{PQ} = \langle 5,2 \rangle - \langle 2,-2 \rangle = \langle 3,4 \rangle$$
$$\hat{v} = \widehat{PQ} = \frac{\langle 3,4 \rangle}{5} \quad \text{(3-4-5 triangle!)}$$

$$D_{\hat{v}} f(2,-2) = \vec{\nabla} f(2,-2) \cdot \hat{v}$$
$$= 2\langle 1,3 \rangle \cdot \frac{\langle 3,4 \rangle}{5} = \frac{2}{5}\underbrace{(3+12)}_{15} = \boxed{6}$$

d) $$f(2,-2) = 2(-2)e^{-2+2} = -4$$
$$\boxed{xye^{-x-y} = -4}$$ level curve thru P.

$$\vec{n} = \langle 1,3 \rangle \propto \vec{\nabla} f(2,-2), \quad \vec{r_0} = \langle 2,-2 \rangle$$
$$0 = \vec{n} \cdot (\vec{r} - \vec{r_0}) = \langle 1,3 \rangle \cdot \langle x-2, y+2 \rangle$$
$$= (x-2) + 3(y+2) = x + 3y \underbrace{-2+6}_{+4}$$
$$\boxed{x + 3y = -4} \quad \text{or} \quad y = \frac{-4-x}{3}$$

e) $$\boxed{L(x,y) = f(2,-2) + f_x(2,-2)(x-2) + f_y(2,-2)(y+2)}$$
$$\boxed{= -4 + 2(x-2) + 6(y+2)}$$

$$z = L(x,y) = -4 + 2x - 4 + 6y + 12 = 4 + 2x + 6y$$
$$\boxed{-2x - 6y + z = 4}$$ tangent plane at P.

② $Y = kNPe^{-N-P}$ $\quad k>0, N>0, P>0$

$$dY = \frac{\partial Y}{\partial N}dN + \frac{\partial Y}{\partial P}dP$$
$$= kP(1-N)e^{-N-P}dN$$
$$+ kN(1-P)e^{-N-P}dP$$
$$= \boxed{ke^{-N-P}(P(1-N)dN + N(1-P)dP)}$$

$$0 = \frac{\partial Y}{\partial N} = kP(1-N)e^{-N-P} \to N=1$$
$$0 = \frac{\partial Y}{\partial P} = kN(1-P)e^{-N-P} \to P=1$$
critical point $\boxed{(1,1).}$ $\boxed{Y(1,1) = ke^{-2}}$

c) $$\frac{\partial^2 Y}{\partial N^2} = k(-P - P(1-N))e^{-N-P}$$
$$= kP(N-2)e^{-N-P}$$
$$\frac{\partial^2 Y}{\partial P^2} = k(-N - N(1-P))e^{-N-P}$$
$$= kN(P-2)e^{-N-P}$$
$$\frac{\partial^2 Y}{\partial P \partial N} = k((1-N))(1-P)e^{-N-P}$$

$$\left.\frac{\partial^2 Y}{\partial N^2}\right|_{(1,1)} = k(-1)e^{-2} < 0 \quad \frown$$
$$\left.\frac{\partial^2 Y}{\partial P^2}\right|_{(1,1)} = k(-1)e^{-2} < 0 \quad \frown$$
$$\left.\frac{\partial^2 Y}{\partial P \partial N}\right|_{(1,1)} = 0 \quad \text{yeah!}$$
local maxes

$$\left.\left(\frac{\partial^2 Y}{\partial N^2}\frac{\partial^2 Y}{\partial P^2} - \left(\frac{\partial^2 Y}{\partial P \partial N}\right)^2\right)\right|_{(1,1)} > 0$$
confirms local max in all directions.

③ $V = xyz = 32 \to z = 32/xy$

$$A = xy + 2\left(\frac{32}{xy}\right)(x+y)$$
$$= xy + 64(x^{-1} + y^{-1}) \equiv f(x,y)$$
$$\frac{\partial A}{\partial x} = y + 64(-x^{-2}) = 0 \to y = 64/x^2$$
$$\frac{\partial A}{\partial y} = x + 64(-y^{-2}) = 0$$
$$0 = x - 64\left(\frac{x^2}{64}\right)^2 = x - \frac{x^4}{64}$$
$$= x\left(1 - \frac{x^3}{64}\right) \to x = (64)^{1/3} = (2^6)^{1/3} = 4$$
$$y = 64/4^2 = 4, \quad z = \frac{32}{4 \cdot 4} = 2$$

dimensions: $\boxed{\text{base } 4, 4 \quad \text{height } 2}$ $\quad$ check on work?

volume: $V = (4)(4)(2) = \boxed{32}$ oops was given, stupid bob

$$(A = 4(4) + 64(\tfrac{1}{4} + \tfrac{1}{4}) = 48)$$
should have asked for this!