MAT2500-01/04 17S Quiz 0.   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

a) Find the distance between the two spheres: $x^2+y^2+z^2=4,\ x^2+y^2+z^2=4x+4y+4z-11$ [redacted].
[Hint: not the distance between the centers!]
b) What fraction of the distance between the centers does this represent?
c) Take your best shot at drawing a 2 dimensional diagram corresponding to any plane through the axis of symmetry through the two centers cutting through the two spheres, labeling the radii and the separation distance as well as the separation between their centers.
d) Hand draw on a set of three axes the rectangular box whose extreme corner from the origin is the center of the second sphere, clearly labeling the axes and their tickmarks.
e) Is the point $\left(\frac{3}{2},\frac{3}{2},\frac{3}{2}\right)$ inside either of the two spheres?
[Hint: what are the distances of this point from the centers?]

▶ solution

a) sphere 1: $C_1(0,0,0),\ r_1=2$

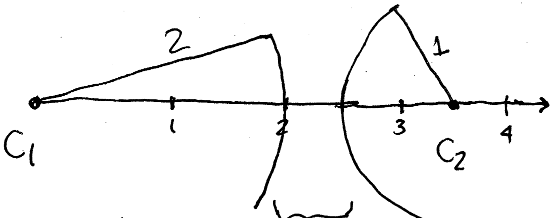
sphere 2: $x^2-4x+y^2-4y+z^2-4z+11=0$

$(x-2)^2-4+(y-2)^2-4+(z-2)^2-4+11=0$

$(x-2)^2+(y-2)^2+(z-2)^2=12-11=1$

$C_2(2,2,2),\ r_2=1$

$|\overrightarrow{C_1C_2}|=\sqrt{2^2+2^2+2^2}=2\sqrt{3}\approx 3.464$
$=\sqrt{12}$ ⟶ always simplify



must subtract radii from center separation

$d=|\overrightarrow{C_1C_2}|-r_1-r_2=2\sqrt{3}-2-1$

$=$ $\boxed{2\sqrt{3}-3\approx 0.464}$

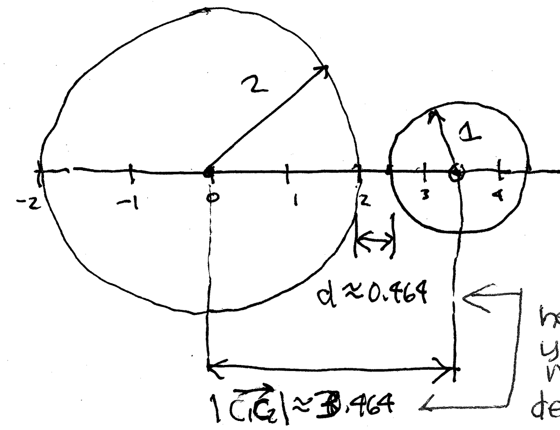
b) $\dfrac{d}{|C_1C_2|}=\boxed{\dfrac{2\sqrt{3}-3}{2\sqrt{3}}}\left(=\dfrac{2-\sqrt{3}}{2}=1-\dfrac{\sqrt{3}}{2}\right)$

≈ 0.13397

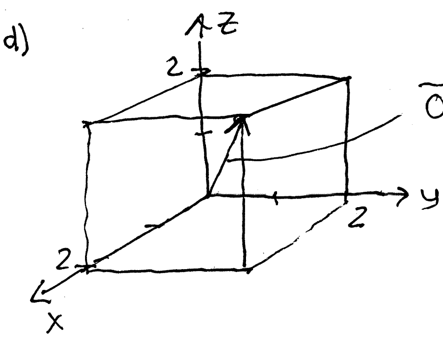
$\boxed{\approx 0.134}\approx\boxed{13.4\%}$

decimal equivalents are needed for interpretation

c)



d)



$\overrightarrow{OC_2}=\langle 2,2,2\rangle$

e) $P(3/2,3/2,3/2)$

$|\overrightarrow{C_1P}|^2=(3/2)^2+(3/2)^2+(3/2)^2=3\left(\frac{9}{4}\right)=\frac{27}{4}$

$|\overrightarrow{C_1P}|=\frac{3}{2}\sqrt{3}\approx 2.598>2$ outside sphere 1

$|\overrightarrow{C_2P}|^2=\left(\frac{3}{2}-2\right)^2+\left(\frac{3}{2}-2\right)^2+\left(\frac{3}{2}-2\right)^2=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2$

$=3/4$

$|\overrightarrow{C_2P}|=\frac{\sqrt{3}}{2}\approx 0.866<1$ $\boxed{\text{inside sphere 2}}$

↑ need decimal equivalents to compare to radii