MAT1505-03/04 17F Quiz 8 Print Name (Last, First) ________________ , ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Is $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}}{2+3n}$ absolutely convergent, conditionally convergent or divergent? Justify your claim.

2. Find the radius of convergence and the interval of convergence (use interval notation) for the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(2x-1)^n}{\sqrt{n}\,5^n}$. Justify your claims. Don't forget the endpoints!

▶ solution

① $$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(n+1)^{1/2}}{2+3(n+1)}}{\frac{n^{1/2}}{2+3n}} = \frac{(n+1)^{1/2}}{n^{1/2}}\left(\frac{2+3n}{5+3n}\right) = \left(\frac{n+1}{n}\right)^{1/2}\left(\frac{\frac{2}{n}+3}{\frac{5}{n}+3}\right) = \left(1+\frac{1}{n}\right)^{1/2}\left(\frac{\frac{2}{n}+3}{\frac{5}{n}+3}\right) \xrightarrow{n\to\infty} 1\cdot\frac{3}{3} = 1$$ ratio test fails.

$$|a_n| = \frac{n^{1/2}}{2+3n} \xrightarrow[n\to\infty]{\text{as}} \frac{n^{1/2}}{3n} = \frac{1}{3n^{1/2}}$$ ~ divergent p-series ($p<1$)

so not absolutely convergent.

But alternating series and nth term (in abs value) decreases to zero, so converges, therefore it is $\boxed{\text{conditionally convergent}}$

② $$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{|2x-1|^{n+1}}{(n+1)^{1/2}5^{n+1}}}{\frac{|2x-1|^n}{n^{1/2}5^n}} = |(2x-1)|\frac{5^n}{5^{n+1}}\frac{n^{1/2}}{(n+1)^{1/2}} = \frac{|2x-1|}{5}\left(\frac{n}{n+1}\right)^{1/2} = \frac{|2x-1|}{5}\frac{1}{\left(1+\frac{1}{n}\right)^{1/2}} \xrightarrow{n\to\infty} \frac{|2x-1|}{5} < 1$$

$$|2x-1| < 5$$

endpoints:

$\frac{|2x-1|}{5} = 1$ $\quad |2x-1| = 5$ $\quad 2x-1 = \pm 5$ $\quad 2x-1 = \begin{cases} 5 \to 2x=6 \to x=3 \\ -5 \to 2x=-4 \to x=-2 \end{cases}$

$|x-\frac{1}{2}| = \boxed{5/2 = R}$

$x=-2$: $\frac{2x-1}{5} = -1$: $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/2}}$ ~ alternating $p=\frac{1}{2}$ series, converges

$x=3$: $\frac{2x-1}{5} = 1$: $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$ ~ $p=\frac{1}{2}$ series, diverges

interval of convergence : $\boxed{[-2,3)}$ or $-2 \le x < 3$

check: R is half width of interval:

interval: $\frac{1}{2} \pm \frac{5}{2} = \begin{cases} 3 \\ -2 \end{cases}$ ✓