MAT1505-03/04 17F Quiz 5   Print Name (Last, First) ________________, ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approxim ations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Consider the catenary arch model function $f(x) = 200 - 20\cosh\left(\frac{x}{30}\right) = 200 - 10\cdot\left(e^{\frac{x}{30}} + e^{-\frac{x}{30}}\right)$ on the interval $-b \le x \le b$, where $f(b) = 0$. This is a slightly simplified St Louis arch.

[Note. If you wish you may use the notation $\cosh(x) = \frac{1}{2}\cdot(e^x + e^{-x})$, $\sinh(x) = \frac{1}{2}\cdot(e^x - e^{-x})$, noting that each is both the derivative and antiderivative of the other.]

a) Use the condition $f(b) = 0$ to determine the endpoint value $b > 0$ exactly in terms of the arccosh function, and then numerically to 8 significant digits.

b) Make a rough plot of the graph of the function f on the given interval, with equal unit tickmarks on the two axes. Label the intercepts. Sketch it roughly here. Draw in the two secant lines between each horizontal intercept and the central maximum on the vertical axis.

c) Evaluate the numerical length of one of these secant lines and double it to get a lower bound for the arclength of the arch, to the nearest integer.

d) Write down a simplified definite integral for the total length of this arch.

e) Now evaluate this numerically, giving the result to 4 decimal places.

f) Comparing to part c), and your plot, does this seem reasonable?

▶ solution

a) $0 = 200 - 20\cosh\left(\frac{x}{30}\right)$

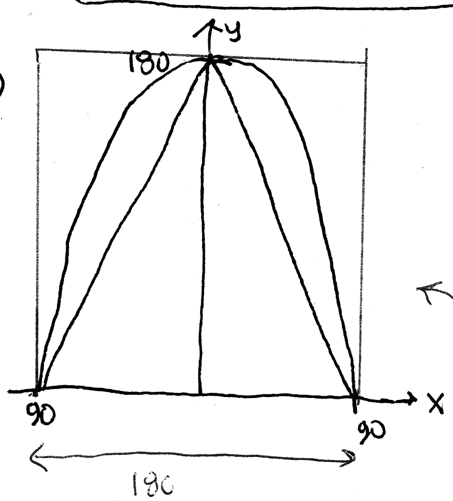
$20\cosh\frac{x}{30} = 200$

$\cosh\frac{x}{30} = \frac{200}{20} = 10$

$\frac{x}{30} = \operatorname{arccosh} 10$

$x = \boxed{30 \operatorname{arccosh} 10 \equiv b \approx 89.796685}$

b)



should be about a square

c) $\ell = 2\sqrt{(180)^2 + b^2} \approx 402$

d) $L = 2\int_0^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$y = 200 - 20\cosh\frac{x}{30}$

$\frac{dy}{dx} = 0 - 20\left(\sinh\frac{x}{30}\right)\left(\frac{1}{30}\right) = -\frac{2}{3}\sinh\frac{x}{30}$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4}{9}\sinh^2\frac{x}{30}$

$\boxed{L = 2\int_0^b \sqrt{1 + \frac{4}{9}\sinh^2\frac{x}{30}}\, dx}$

e) $\overset{\text{Maple}}{\approx} \boxed{430.4207}$

f) It is a bit bigger as it should be.