Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Given the velocity profile $v(r) = \dfrac{P}{4\,\eta\,\ell}\,(R^2 - r^2)$ for $0 \leq r \leq R$ for blood flow in a long cylindrical tube, a more appropriate average would be one weighted by the differential area $dA = 2\,\pi\,r\,dr$ represented by the annular strip of thickness dr, namely

$$\langle v \rangle = \frac{\displaystyle\int_0^R v(r)\,2\,\pi\,r\,dr}{\displaystyle\int_0^R 2\,\pi\,r\,dr} = \frac{\displaystyle\int_0^R v(r)\,2\,\pi\,r\,dr}{\pi\,R^2}.$$

a) For the given velocity profile, introduce the change of variable $x = \dfrac{r}{R}$ into this integral expression to re-express it completely in terms of the new dimensionless variable x and re-express the coefficient of the resulting integral (don't evaluate the integral yet!) in terms of the maximum velocity $v_{max} = v(0)$.

b) Now integrate by hand the resulting x variable integral, giving your final result as a multiple of the maximum velocity.

c) At what fraction of the radius R does the velocity equal this new average velocity?

2. a) Rotate the region below the graph $y = f(x) = x\,e^{-x}$, $0 \leq x \leq 1$ about its maximum value $y_{max} = e^{-1}$ [verify this claim that it is the maximum] and set up an integral representing this volume of revolution and simplify the integrand. Support your choice of integrand with a fully labeled diagram labeling the starting and stopping variable value equations of a typical cross-section, shading the relevant integration region with equally spaced such cross-sections, and identifying relevant disk radii in the diagram.

b) Evaluate it exactly with technology.

c) To the nearest percent, what percentage does this represent of the volume of the cylinder obtained by rotating the entire rectangular region between the horizontal line $y = y_{max}$ and the horizontal axis around that same axis $y = y_{max}$?

3. a) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} t\,\sin(2\,t)\,dt$ exactly with technology.

b) Show the evaluation by hand using integration by parts, step by step, not mixing indefinite and definite integral notation.

▶ **solution**

▼ **pledge**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: