MAT1505-03/04 17F Quiz 0 Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Consider the function $f(x) = 1 + 2x - e^x$ whose graph is shown in the figure.

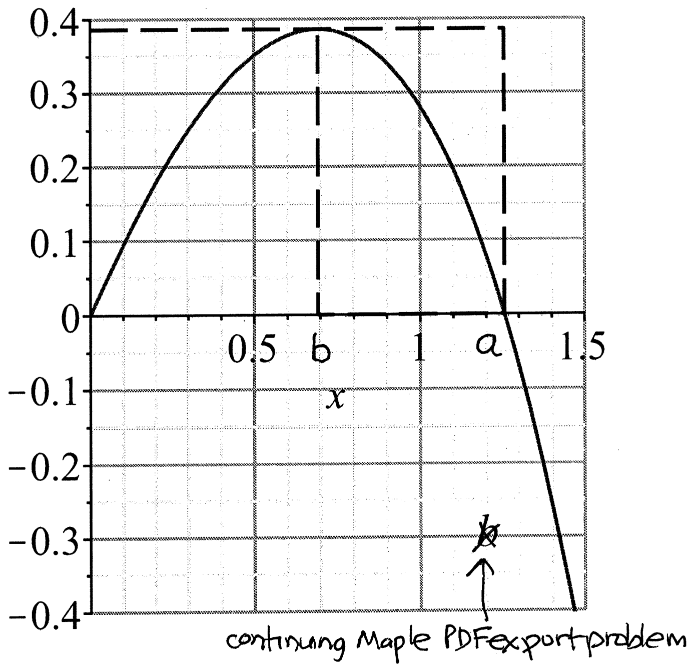
a) Find the value of b exactly and approximately to 6 decimal places; repeat for $f(b)$.

b) Find the value of a exactly and approximately to 6 decimal places (Hint: Maple can easily solve the necessary equation.)

c) What is the area of the larger dashed box to 6 decimal places? What percentage of this area lies under the curve inside this box, making a rough guess?

d) Evaluate the exact area under the curve above the horizontal axis, and approximately to 6 decimal places.

e) Evaluate the percentage of the box area this represents, to the nearest integer percent. Does this seem reasonable? Was your initial guess close?



solution

a) $f(x) = 1+2x-e^x$

$f'(x) = 0+2-e^x = 0$

$\ln[e^x = 2] \to \boxed{x = \ln 2 \approx 0.693147} \boxed{=b!}$

$\boxed{f(\ln 2) =} \; 1+2\ln 2 - \underbrace{e^{\ln 2}}_{=2}$

$= 2\ln 2 - 1$

$\boxed{\approx 0.386294} \boxed{=f(b)!}$

b) Maple says:

$\boxed{a = -\frac{1}{2}\text{LambertW}\left(-1, -\frac{1}{2}e^{-\frac{1}{2}}\right) - \frac{1}{2} \approx 1.256431}$

c) $\boxed{A_{box} = a\, f(b) \approx 0.485352}$

d) $A = \int_0^a f(x)\,dx = \int_0^a 1+2x-e^x\,dx$

$= x + x^2 - e^x \Big|_0^a \boxed{= a + a^2 - e^a \approx 0.322188}$

e) $\frac{A}{A_{box}} \approx 0.663823 \approx 0.66$

$\to 66\%$

A is clearly more than half the box area (~~formed by~~ the triangle with vertices $(0,0)-(0,a)-(b,f(b))$ has half the box area) but not a whole lot more. I would have guessed roughly 2/3 if I had followed my own instructions.

66% is right on the money!

So yes, it is reasonable, and very close to my initial guess, had I guessed initially.

:-)

exact / numerical approximation

$e \approx 2.718$

$\pi \approx 3.14159$

$\sqrt{2} \approx \underbrace{1.414}$

4 significant digits, 3 decimal places