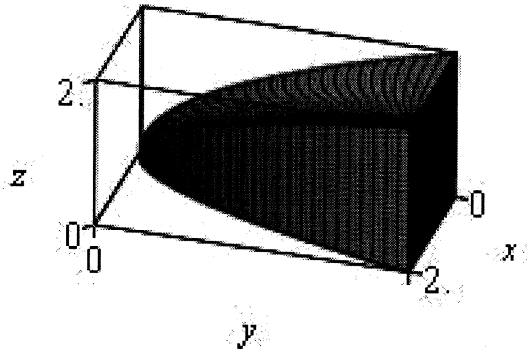
MAT2500-01/04 16S Quiz 7 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).



Consider the solid region R in the first octant enclosed by the surfaces $z=\sqrt{y}$, $y=x^2$, $y=4$, $x=0$ and the x-y plane.

a) Set up an iterated triple integral $V=\iiint 1\ dV$ for the volume of this region in the order $dV=dx\ dz\ dy$ and give its exact value using Maple. Support your limits of integration with a diagram for the outer double integral with a typical completely labeled line segment cross-section (bullet endpoints labeled by start and stop equations, arrow in increasing variable direction) and equally spaced such cross-sections for the shading, and a similar diagram for the innermost integral.

b) Repeat with new such diagrams for the order $dV=dz\ dy\ dx$.

Optional. Can you set up a sum of two integrals that give the same result for the order $dy\ dz\ dx$?

solution

looking at the figure:

a) top: $z=\sqrt{y}$

front side: $y=x^2 \rightarrow x=\sqrt{y}$ $(x\geq 0)$

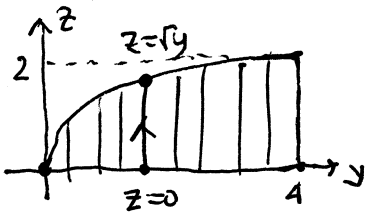
back side: $x=0$

right side: $y=4$

bottom: $z=0$

x first: $x=0..\sqrt{y}$

project onto yz plane:



z next: $z=0..\sqrt{y}$

y last: $y=0..4$

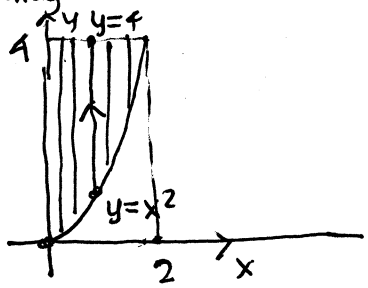
so:

$$V=\int_0^4\int_0^{\sqrt{y}}\int_0^{\sqrt{y}} 1\ dx\,dz\,dy$$

Maple 8

b) z first: $z=0..\sqrt{y}$ as in left projection diagram

y next:

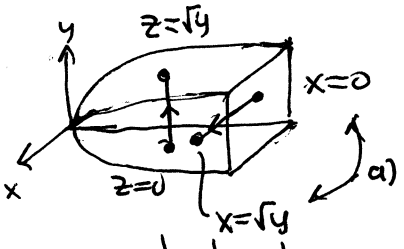


x last: $x=0..2$

so

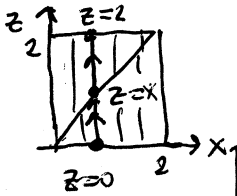
$$V=\int_0^2\int_{x^2}^4\int_0^{\sqrt{y}} 1\ dz\,dy\,dx$$

Maple 8



innermost integral diagrams

optional. $z=\sqrt{y} \rightarrow y=z^2$ top intersects front side $y=x^2$ at $z^2=x^2 \rightarrow z=x$.



top triangle: $y=z^2..4$ while $z=x..2$ as $x=0..2$

bot triangle: $y=x^2..4$ while $z=0..x$ as $x=0..2$

$$V=\int_0^2\int_x^2\int_{z^2}^4 1\,dy\,dz\,dx+\int_0^2\int_0^x\int_{x^2}^4 1\,dy\,dz\,dx$$