MAT2500-01/04 16S Quiz 6 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

Consider the triangular region R of the plane with vertices: $(0,0)$, $(4,0)$, $(4,3)$. Let $M=\iint_R x\,dA$.

a) Draw the region of integration and fill it with equally spaced cross-sections appropriate for the integration order $\iint ... dy\,dx$. Then annotate a typical such linear cross-section by labeling its bullet point endpoints by the starting and stopping value equations of the variable running along the cross-section, and putting with an arrowhead somewhere in the middle of the cross-section indicating increasing values of the integration variable. This justifies your limits of integration for the iterated integral in this integration order. State its expression, but use Maple to evaluate it to a number.

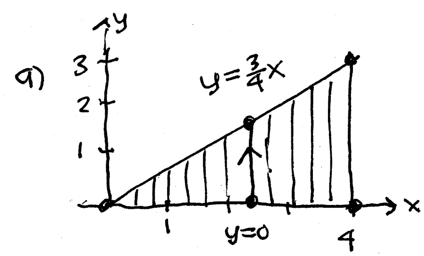
b) Repeat with a new such diagram for the order $\iint ... dx\,dy$.

c) Repeat using polar coordinates with a third such diagram.

d) Now show the step by step evaluation of this polar coordinate iterated integral, showing the antiderivatives and their evaluations.

[Do all three evaluations of this same integral give the same result as they should?]

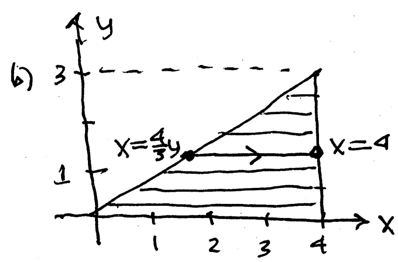
▶ solution

a) 

3-4-5 triangle
slope hypotenuse: 3/4

so $y = 0 .. \frac{3}{4}x$ while $x = 0..4$

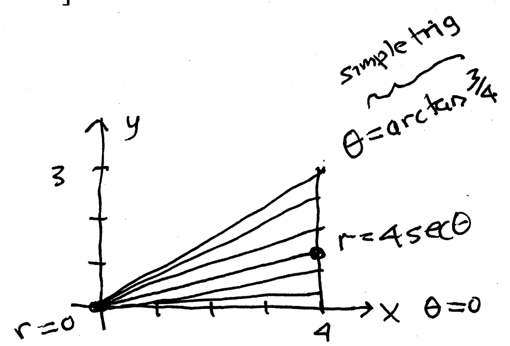
$$\boxed{M = \int_0^4 \int_0^{\frac{3}{4}x} x\,dy\,dx \overset{\text{Maple}}{=} 16}$$

b) 

$y = \frac{3}{4}x \to x = \frac{4}{3}y$

so $x = \frac{4}{3}y .. 4$ while $y = 0..3$

$$\boxed{M = \int_0^3 \int_{\frac{4}{3}y}^{4} x\,dx\,dy \overset{\text{Maple}}{=} 16}$$

c) 

$x = 4 \to r\cos\theta = 4$

$\to r = \frac{4}{\cos\theta} = 4\sec\theta$

so $r = 0..4\sec\theta$ while $\theta = 0..\arctan\frac{3}{4}$

$$\boxed{M = \int_0^{\arctan 3/4} \int_0^{4\sec\theta} (r\cos\theta)\,r\,dr\,d\theta}$$

$$\boxed{=16}$$
Maple

d) $M = \int_0^{\arctan\frac{3}{4}} \underbrace{\int_0^{4\sec\theta} r^2\cos\theta\,dr}\,d\theta$

$\frac{r^3}{3}\cos\theta\Big|_{r=0}^{r=4\sec\theta} = \frac{64\sec^3\theta\,\cos\theta}{3} = \frac{64}{3}\sec^2\theta$

$= \int_0^{\arctan 3/4} \frac{64}{3}\sec^2\theta\,d\theta = \frac{64}{3}\tan\theta\Big|_0^{\arctan 3/4} = \frac{64}{3}\tan\left(\arctan\frac{3}{4}\right) = \frac{64}{3}\left(\frac{3}{4}\right)$

$\frac{d}{d\theta}(\tan\theta)$!
memory or Maple.

$= \boxed{16}$ ✓ yes, all 3 agree