MAT2500-01/04 16S Quiz 5 Print Name (Last, First) ____________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

A sound with frequency f_s is produced by a source traveling along a line with speed v_s. If an observer is traveling with speed v_o along the same line from the opposite direction toward the source, then the frequency of the sound heard by the observer is

$$f_o = \left(\frac{c+v_o}{c-v_s}\right) f_s .$$

where c is the speed of sound, about 332 m/s. (This is the Doppler effect.) Suppose that, at a particular moment, you are in a train traveling at 34 m/s and accelerating at 1.2 m/s^2. A train is approaching you from the opposite direction on the other track at 40 m/s^2, accelerating at 1.4 m/s^2, and sounds its whistle, which has a frequency of 440 Hz. At that instant, what is the perceived frequency that you hear and how fast is it changing?

a) State the values of $c, f_s; v_o, v_o', v_s, v_s'$.

b) Numerically evaluate f_o.

c) Use the chain rule to evaluate df_o/dt as a function of $c, f_s, v_o, v_o', v_s, v_s'$, combining the terms into a single fraction.

d) Evaluate this formula numerically.

e) Answer the question posed by the word problem in a complete English sentence with correct units and with some thought to appropriate significant digits.

f) **Optional.** Searching the web for a few seconds, one finds that the whistle frequency f_s corresponds to the musical note A_4. Which musical tone is closest to the perceived frequency at this moment, and how long would it take at this rate to reach it exactly? **Post-quiz:** assuming constant accelerations over this time interval, calculate the final values of the two speeds and use them to evaluate the actual perceived frequency at that new moment. Note that this value nails that nearest musical tone.

▶ solution

"about 332 m/s" 3 sig figs. but 2 sig figs on vel, acceleration, 3 or 4 sig figs? 3 for the word problem response because musical notes... well, they change in first 2 sig figs... so here 2 sig figs would also be okay.

a) $c = 332,\ f_s = 440,\ v_o = 34,\ v_o' = 1.2,\ v_s = 40,\ v_s' = 1.4$

b) $f_o\big|_{t_0} = \dfrac{332+34}{332-40}\,440 = \dfrac{366}{292}\,440 \overset{\text{Maple}}{=} 551.51 \approx \boxed{551.5} \ \boxed{\approx 552}$

c) $\boxed{\dfrac{df_o}{dt}} = \dfrac{\partial f}{\partial v_o} v_o' + \dfrac{\partial f}{\partial v_s} v_s' = \left(\dfrac{\partial}{\partial v_o}\left(\dfrac{c+v_o}{c-v_s}\right) v_o' + \dfrac{\partial}{\partial v_s}\left((c+v_o)(c-v_s)^{-1}\right) v_s'\right) f_s$

$= \left(\dfrac{1}{c-v_s} v_o' - (c+v_o)(c-v_s)^{-2}(0-1)\, v_s'\right) f_s = \left(\dfrac{v_o'}{c-v_s} + \dfrac{c+v_o}{(c-v_s)^2} v_s'\right) f_s$

$= \boxed{\dfrac{(c-v_s)v_o' + (c+v_o)v_s'}{(c-v_s)^2} f_s}$

note: we could have gotten this result directly from Calc I knowledge & the quotient rule – just a related rates problem.

d) $\dfrac{df_o}{dt}\Big|_{t_0} = \dfrac{(332-40)1.2 + (332+34)1.4}{(332-40)^2}(440) \overset{\text{Maple}}{=} \boxed{4.452}\ \boxed{\approx 4.45}\ \boxed{\approx 4.5}$

e) The perceived frequency is about 552 Hz and it is increasing at 4.45 Hz/s at that moment. (2 sig figs also acceptable)

f) 551.5 Hz is just a bit short of 554.37 which corresponds to D_5 flat.

Then $T = (554.37 - 551.5)/4.452 = 0.643$ sec

But then $v_o \to v_o + v_o' T = 34.77$, $v_s \to v_s + v_s' T = 40.90$ and the new value of f_o is 554.379 $\approx$ 554.38. Dead on.