MAT2500-01/04 16s Quiz 4 Take Home Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x, y) = x\,e^{2xy}$

a) Evaluate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$.

b) Evaluate both $\frac{\partial^2 f}{\partial y\,\partial x}$ and $\frac{\partial^2 f}{\partial x\,\partial y}$ in the orders indicated, factoring out the exponential, and evaluate their common value at $(1, 0)$.

2. Determine whether or not $u = e^{-x}\cos(y) - e^{-y}\cos(x)$ is a solution of Laplace's equation $u_{xx} + u_{yy} = 0$.

3. The wave height in feet function $h = h(v, t)$ depends on the speed v of the wind in knots and the length of time in hours that the wind has been blowing at that speed. Describe in words the meaning of the question "For what value of v is $h(v, 20) = 28$ (i.e., translate this question into words in a complete sentence).

▶ solution

① a) $f(x,y) = xe^{2xy}$

$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}(xe^{2xy}) = 1\cdot e^{2xy} + x\,e^{2xy}\frac{\partial}{\partial x}(2xy)$

$= e^{2xy} + 2xye^{2xy}$

$= \boxed{e^{2xy}(1+2xy)}$

$\frac{\partial f}{\partial y}(x,y) = \frac{\partial}{\partial y}(xe^{2xy}) = x\,e^{2xy}\frac{\partial}{\partial y}(2xy)$

$= xe^{2xy}(2x) = \boxed{2x^2e^{2xy}}$

b) $\frac{\partial^2 f}{\partial y\partial x}(x,y) = \frac{\partial}{\partial y}(e^{2xy}(1+2xy))$

$= e^{2xy}\frac{\partial}{\partial y}(2xy)(1+2xy) + e^{2xy}\frac{\partial}{\partial y}(1+2xy)$

$= e^{2xy}\,2x(1+2xy) + e^{2xy}(2x)$

$= e^{2xy}(2x)(2+2xy) = \boxed{e^{2xy}4x(1+xy)}$

$= \boxed{e^{2xy}(4x + 4x^2y)}$

$\frac{\partial^2 f}{\partial x\partial y}(x,y) = \frac{\partial}{\partial x}(2x^2e^{2xy}) = 4xe^{2xy} + 2x^2e^{2xy}\frac{\partial}{\partial x}e^{2xy}$

$= 4xe^{2xy} + 4x^2y\,e^{2xy}$

$= \boxed{e^{2xy}(4x+4x^2y)}$

$\frac{\partial^2 f}{\partial x\partial y}(1,0) = e^0(4+0) = \boxed{4}$

② $u = e^{-x}\cos y - e^{-y}\cos x$

$u_x = -e^{-x}\cos y + e^{-y}\sin x$

$u_{xx} = e^{-x}\cos y + e^{-y}\cos x$

$u_y = -e^{-x}\sin y + e^{-y}\cos x$

$u_{yy} = -e^{-x}\cos y - e^{-y}\cos x$

$u_{xx} + u_{yy} = e^{-x}\cos y + e^{-y}\cos x + -e^{-x}\cos y - e^{-y}\cos x$

$= 0$ ✓

yes, it is a soln.

③ For what wind speed is the wave height 28 feet after it has been blowing for 20 hours?