Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(3, 3, 2)$, $P_2(-1, 1, 2)$, $P_3(2, 2, 4)$ and the parallelopiped formed from their three position vectors $\vec{r_1}, \vec{r_2}, \vec{r_3}$. [Note $\vec{r_1}, \vec{r_3}$ form the front face of this object, while $\vec{r_2}$ points back.]
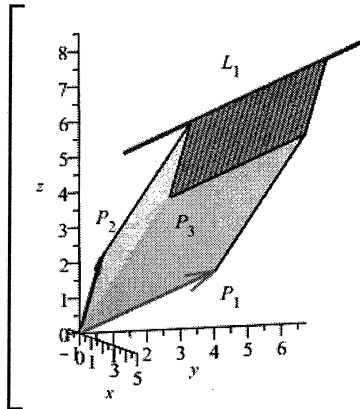
a) Write the parametrized equations of the line L_1 through the back edge of the top face $\mathscr{P}_{topface}$ (shaded) as shown in this view. [What is the simplest (smallest integer components) position vector of a point on the line to use to write its equation? What is the orientation of the line?] Where does this line intersect the $y = 0$ plane?

b) Find a normal vector $\vec{n}$ for the plane $\mathscr{P}_{topface}$ which contains the top face of the parallelepiped shown in the figure.

c) Write the simplified equation for this plane. Does the point at the tip of the main diagonal of the parallelopiped (from the origin to the opposite corner) satisfy this equation as it should?

d) Find the scalar projection h of the main diagonal of the parallelopiped along $\vec{n}$. [$|h|$ is just the distance of the top face plane from the origin, or its height if we instead think of that face as the top of the parallelopiped.]

e) Evaluate the area A of the top face of the parallelopiped, a parallelogram formed by the edges parallel to $\vec{r_1}, \vec{r_2}$.

f) Does the volume $V = |h| A$ of the parallelopiped equal the triple scalar product $|\vec{r_3} \cdot (\vec{r_1} \times \vec{r_2})|$ as it should?

a) $\vec{r_2} + \vec{r_3}$ is the left vertex on L_1 while $\vec{r_2} + \vec{r_3} + \vec{r_1}$ is the right vertex, either can be used to write the eqn of the line but the first is simpler:

$\vec{r_2} + \vec{r_3} = \langle -1, 1, 2 \rangle + \langle 2, 2, 4 \rangle = \langle -1+2, 1+2, 2+4 \rangle = \langle 1, 3, 6 \rangle \equiv \vec{r_0}$

The line orientation is along $\vec{r_1} = \langle 3, 3, 2 \rangle$ so

$\vec{r} = \langle x, y, z \rangle = \vec{r_0} + t\vec{r_1} = \langle 1, 3, 6 \rangle + t\langle 3, 3, 2 \rangle = \langle 1+3t, 3+3t, 6+2t \rangle$

so $\boxed{x = 1+3t, \; y = 3+3t, \; z = 6+2t}$ then $0 = y = 3+3t \to t = -1 \to x = 1+3(-1) = -2; \; z = 6+2(-1) = 4$

so it intersects the $y = 0$ plane at the point $\boxed{(-2, 0, 4)}$.

b) The edges of the top face are along $\vec{r_1}$ and $\vec{r_2}$ so $\vec{n} = \vec{r_1} \times \vec{r_2} = \langle 3, 3, 2 \rangle \times \langle -1, 1, 2 \rangle \overset{Maple}{=} \langle 4, -8, 6 \rangle$

$= 2\langle 2, -4, 3 \rangle$ $\vec{n}$ simpler

c) $0 = \vec{n} \cdot (\vec{r} - \langle 1, 3, 6 \rangle) = \langle 2, -4, 3 \rangle \cdot \langle x-1, y-3, z-6 \rangle$

$= 2(x-1) - 4(y-3) + 3(z-6) = 2x - 4y + 3z - 2 + 12 - 18 = \boxed{2x - 4y + 3z - 8 = 0}$

or $\boxed{2x - 4y + 3z = 8}$

Check: $\vec{r_1} + (\vec{r_2} + \vec{r_3}) = \langle 3, 3, 2 \rangle + \langle 1, 3, 6 \rangle = \langle 4, 6, 8 \rangle$

$2(4) - 4(6) + (8) = 8 - 24 + 24 = 8 \; \checkmark$ yes!

d) $|\vec{n}| = \sqrt{2^2 + (-4)^2 + 3^2} = \sqrt{4 + 16 + 9} = \sqrt{29} \to \hat{n} = \langle 2, -4, 3 \rangle / \sqrt{29}$

$\hat{n} \cdot (\vec{r_1} + \vec{r_2} + \vec{r_3}) = \frac{1}{\sqrt{29}} \langle 2, -4, 3 \rangle \cdot \langle 2, 3, 4 \rangle \cdot 2$

$= \frac{2(4 - 12 + 12)}{\sqrt{29}} = \boxed{\frac{8}{\sqrt{29}} = h}$

e) $A = |\vec{r_1} \times \vec{r_2}| = |2\langle 2, -4, 3 \rangle| = 2|\vec{n}| = \boxed{2\sqrt{29} = A}$

f) $V = hA = \frac{8}{\sqrt{29}}(2\sqrt{29}) = \boxed{16}$

$\vec{r_3} \cdot (\vec{r_1} \times \vec{r_2}) = 2\langle 1, 1, 2 \rangle \cdot 2\langle 2, -4, 3 \rangle = 4(2 - 4 + 6) = \boxed{16} \; \checkmark$ yes, $|\vec{r_3} \cdot (\vec{r_1} \times \vec{r_2})| = V$

$2\vec{n}$