MAT2500-01/04 16S Quiz 0. Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given the two points $A(-1, 5, 3)$, $B(6, 2, -2)$

a) Take your best shot at drawing these two points on a system of Cartesian axes using rectangular coordinate boxes to locate each point with respect to those axes. Be sure you label each axis with its variable name and label their tickmarks.

b) The distance of the point $P(x, y, z)$ from point A is twice the distance of P from point B. Show that the set of all such points P is a sphere, and write down the standard equation for this sphere identifying its center $C(x_0, y_0, z_0)$ and radius a.

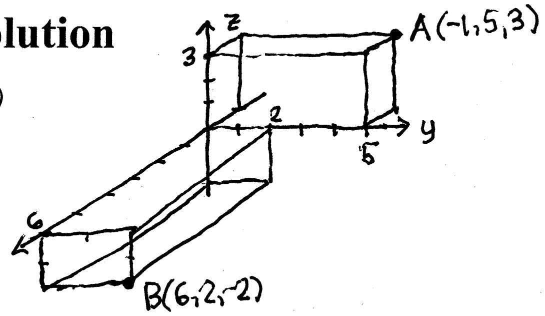
c) Are the three points A, B, C collinear? If so, demonstrate this property.

d) Which is the lowest point? Why?

e) Which of the first two points lies inside the sphere? Why?

▶ solution

a)



b) $|\vec{AP}| = 2|\vec{BP}|$ so $|\vec{AP}|^2 = 4|\vec{BP}|^2$

$P(x,y,z)$:

$(x-(-1))^2+(y-5)^2+(z-3)^2 = 4\left[(x-6)^2+(y-2)^2+(z-(-2))^2\right]$

$(x+1)^2+(y-5)^2+(z-3)^2 = 4\left[(x-6)^2+(y-2)^2+(z+2)^2\right]$

$x^2+2x+1 \quad = 4(x^2-12x+36)$
$+y^2-10y+25 \quad +4(y^2-4y+4)$
$+z^2-6z+9 \quad +4(z^2+4z+4)$

$3x^2-(48+2)x+144-1$
$+3y^2+(-16+10)y+16-25$
$+3z^2+(16+6)z+\underbrace{16-9}_{141}$
$\rightarrow 3(x^2-\frac{50}{3}x) + 141 = 0$
$+3(y^2-2y)$
$+3(z^2+\frac{22}{3}z)$

$(x-\frac{25}{3})^2-(\frac{25}{3})^2 + \frac{141}{3} = 0$
$+(y-1)^2 - 1^2$
$+(z+\frac{11}{3})^2-(\frac{11}{3})^2$
$\underbrace{\qquad\qquad}_{-\frac{332}{9}}$ (Maple)

compare with: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = a^2$

so $\boxed{a = \frac{\sqrt{332}}{3} = \frac{2\sqrt{83}}{3}} \approx 6.074$

$\boxed{(x_0,y_0,z_0) = (\frac{25}{3}, 1, -\frac{11}{3})} \approx (8.33, 1, -3.67)$

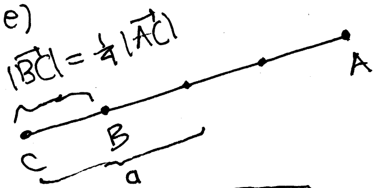
c) $\vec{AC} = \langle \frac{25}{3}+1, 1-5, -\frac{11}{3}-3\rangle = \langle \frac{28}{3}, -4, -\frac{20}{3}\rangle$

$\vec{BC} = \langle \frac{25}{3}-6, 1-2, -\frac{11}{3}+2\rangle = \langle \frac{7}{3}, -1, -\frac{5}{3}\rangle$

obviously $\vec{AC} = 4\vec{BC}$ so $\boxed{A, B, C \text{ collinear}}$

d) C has the lowest value of z so C is the lowest point

e) $|\vec{BC}| = \frac{1}{4}|\vec{AC}|$



$|\vec{BC}| = \sqrt{(\frac{7}{3})^2+(-1)^2+(\frac{5}{3})^2} = \frac{\sqrt{49+9+25}}{3} = \frac{\sqrt{83}}{3}$

$= \frac{1}{2}a < a$ so $\boxed{B \text{ is inside sphere.}}$

and $|\vec{AC}| = 4(\frac{1}{2}a) = 2a$ so A is outside sphere