MAT2500-01/04 16S Test 1 Answers

a) $\vec r = e^t\langle \cos t, \sin t, 1\rangle$

$\vec v = \vec r\,' = e^t\langle \cos t, \sin t, 1\rangle + e^t\langle -\sin t, \cos t, 0\rangle$

$= \boxed{e^t\langle \cos t - \sin t, \sin t + \cos t, 1\rangle}$

$v = |\vec r\,'| = e^t\sqrt{(c-s)^2 + (c+s)^2 + 1}$

$= e^t\sqrt{\underbrace{c^2 - \cancel{2cs} + s^2}_{1} + \underbrace{c^2 + \cancel{2cs} + s^2}_{0} + 1}$

$= \boxed{e^t\sqrt3}$

$\vec a = \vec r\,'' = e^t\langle c-s, c+s, 1\rangle + e^t\langle -s-c, c-s, 0\rangle$

$= e^t\langle -2s, 2c, 1\rangle$

$= \boxed{e^t\langle -2\sin t, 2\cos t, 1\rangle}$

$a = |\vec r\,''| = e^t\sqrt{4\sin^2 t + 4\cos^2 t + 1}$

$= \boxed{\sqrt5\, e^t}$

$\hat T = \dfrac{e^t\langle c-s, c+s, 1\rangle}{\sqrt3\, e^t} = \dfrac{1}{\sqrt3}\langle \cancel{\sin t - \cos t},$

$= \boxed{\dfrac{1}{\sqrt3}\langle \cos t - \sin t, \sin t + \cos t, 1\rangle}$ $\boxed{\hat T(0) = \dfrac{1}{\sqrt3}\langle 1,1,1\rangle}$

$\vec r(0) = \langle 1, 0, 1\rangle$

$\vec r\,'(0) = \langle 1, 1, 1\rangle \qquad |\vec r\,'(0)| = \sqrt3$

$\vec r\,''(0) = \langle 0, 2, 1\rangle \qquad |\vec r\,''(0)| = \sqrt5$

$\hat T(0) = \dfrac{1}{\sqrt3}\langle 1,1,1\rangle$

b) $\vec r = \vec r(0) + t\,\vec r\,'(0)$

$\boxed{\langle x,y,z\rangle = \langle 1,0,1\rangle + t\langle 1,1,1\rangle = \langle 1+t, t, 1+t\rangle}$

note $t=0$! $\vec r = \langle 0,-1,0\rangle$ on y-axis

c) $\vec b = \vec r\,'\times\vec r\,'' = e^t\langle c-s, c+s, 1\rangle \times e^t\langle -2s, 2c, 1\rangle$

$\overset{\text{Maple}}{=} \boxed{e^{2t}\langle \sin t - \cos t, -\sin t - \cos t, 2\rangle}$

$|\vec b| = e^{2t}\sqrt{(s-c)^2 + (-s-c)^2 + 4}$

$= e^{2t}\sqrt{s^2 - \cancel{2cs} + c^2 + s^2 + \cancel{2cs} + c^2 + 4}$

$= \boxed{\sqrt6\, e^{2t}}$

$\boxed{\vec b(0) = \langle -1,-1,2\rangle \qquad |b(0)| = \sqrt6}$

d) $\vec n = \langle -1,-1,2\rangle$, $\vec r_0 = \vec r(0) = \langle 1,0,1\rangle$

$0 = \vec n\cdot(\vec r - \vec r_0) = \langle -1,-1,2\rangle\cdot\langle x-1, y, z-1\rangle$

$= -(x-1) - (y) + 2(z-1) = -x - y + 2z + 1 - 2$

$\boxed{-x-y+2z=1}$

e) $K = \dfrac{|\vec b|}{v^3} = \dfrac{\sqrt6\, e^{2t}}{(\sqrt3\, e^t)^3} = \dfrac{\sqrt6}{3\sqrt3}e^{-t}$

$= \boxed{\dfrac{\sqrt2}{3}e^{-t}}$

$\boxed{\rho = \dfrac{3}{\sqrt2}e^t}$ $\boxed{\rho(0) = \dfrac{3}{\sqrt2}}$

f) $\hat B = \dfrac{\vec b}{|b|} = \dfrac{e^{2t}\langle s-c, -s-c, 2\rangle}{\sqrt6\, e^{2t}}$

$= \boxed{\dfrac{1}{\sqrt6}\langle \sin t - \cos t, -\sin t - \cos t, 2\rangle}$, $\boxed{\hat B(0) = \dfrac{1}{\sqrt6}\langle -1,-1,2\rangle}$

g) $\hat N = \hat B\times\hat T = \dfrac{1}{\sqrt6}\langle s-c, -s-c, 2\rangle \times \dfrac{1}{\sqrt3}\langle c-s, s+c, 1\rangle$

$\overset{\text{Maple}}{=} \dfrac{1}{3\sqrt2}\langle -3(c+s), 3(c-s), 0\rangle = \dfrac{1}{\sqrt2}\langle$

$= \boxed{\dfrac{1}{\sqrt2}\langle -\cos t - \sin t, \cos t - \sin t, 0\rangle}$

h) see below

i) $L = \displaystyle\int_{-\pi}^{\pi} v\,dt = \int_{-\pi}^{\pi}\sqrt3\, e^t\,dt = \sqrt3\, e^t\Big|_{-\pi}^{\pi}$

$= \boxed{\sqrt3(e^{\pi} - e^{-\pi}) \approx 40.0060}$

j) $\vec C(0) = \vec r(0) + \rho(0)\hat N(0)$

$= \langle 1,0,1\rangle + \dfrac{3}{\sqrt2}\dfrac{1}{\sqrt2}\langle -1,1,0\rangle$

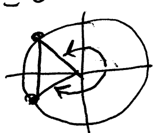
$= \langle 1,0,1\rangle + \langle -\frac32, \frac32, 0\rangle = \boxed{\langle -\frac12, \frac32, 1\rangle}$ ✓

k) $\vec{OSC}(0,\theta) = \vec C(0) + \rho(0)\left(\cos\theta\,\hat T(0) + \sin\theta\,\hat N(0)\right)$

$= \langle -\frac12, \frac32, 1\rangle + \dfrac{3}{\sqrt2}\left(\cos\theta\,\dfrac{1}{\sqrt3}\langle 1,1,1\rangle + \sin\theta\,\dfrac{1}{\sqrt2}\langle -1,1,0\rangle\right)$

$= \langle -\frac12, \frac32, 1\rangle + \cos\theta\,\dfrac{\sqrt3}{\sqrt2}\langle 1,1,1\rangle + \sin\theta\,\dfrac32\langle -1,1,0\rangle$

$\boxed{= \langle -\frac12 + \frac{\sqrt3}{\sqrt2}\cos\theta - \frac32\sin\theta, \frac32 + \frac{\sqrt3}{\sqrt2}\cos\theta + \frac32\sin\theta, 1 + \sqrt{\tfrac32}\cos\theta\rangle = \langle x,y,z\rangle}$ on $-\pi\le\theta\le\pi$ or $0\le\theta\le 2\pi$



$0 = z = 1 + \sqrt{\tfrac32}\cos\theta \to \cos\theta = -\sqrt{\tfrac23} \to$

$\theta = \arccos(-\sqrt{\tfrac23}) = \pi - \arccos\sqrt{\tfrac23}$

or $\pi + \arccos\sqrt{\tfrac23}$

$\sin\theta = \pm\sqrt{1 - \tfrac23} = \pm\dfrac{1}{\sqrt3}$

not going well, backtrack

$\vec{OSC}(0,\theta) = \cancel{\langle -\frac12 - \frac{\sqrt3}{\sqrt2}(-\sqrt{\tfrac23} \pm \tfrac{3}{2\sqrt3}), \frac32 + \frac{\sqrt3}{\sqrt2}(-\sqrt{\tfrac23} \pm \tfrac{1}{\sqrt3}), 0\rangle}$

$= \langle -\frac12, \frac32, 1\rangle + -1\langle 1,1,1\rangle \pm \dfrac{\sqrt3}{2}\langle -1,1,0\rangle$

$= \langle -\frac32, \frac12, 0\rangle \pm \dfrac{\sqrt3}{2}\langle -1,1,0\rangle$

$= \boxed{\langle -\frac32 - \frac{\sqrt3}{2}, \frac12 + \frac{\sqrt3}{2}, 0\rangle, \langle -\frac32 + \frac{\sqrt3}{2}, \frac12 - \frac{\sqrt3}{2}, 0\rangle}$

h) $\vec a(0) = \langle 0,2,1\rangle$, $\hat N(0) = \dfrac{1}{\sqrt2}\langle -1,1,0\rangle$, $\hat T(0) = \dfrac{1}{\sqrt3}\langle 1,1,1\rangle$

$a_T(0) = \hat T(0)\cdot\vec a(0) = \dfrac{1}{\sqrt3}\langle 1,1,1\rangle\cdot\langle 0,2,1\rangle = \dfrac{3}{\sqrt3} = \boxed{\sqrt3}$

$a_N(0) = \hat N(0)\cdot\vec a(0) = \dfrac{1}{\sqrt2}\langle -1,1,0\rangle\cdot\langle 0,2,1\rangle = \dfrac{2}{\sqrt2} = \boxed{\sqrt2}$

$|\vec a(0)|^2 = 5 = 3 + 2$ ✓ check