MAT2705-02/04 16F Quiz 8 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1.
$x_1'(t) = 5\,x_1(t) - 6\,x_2(t) + 3\,x_3(t)$, $x_2'(t) = 6\,x_1(t) - 7\,x_2(t) + 3\,x_3(t)$, $x_3'(t) = 6\,x_1(t) - 6\,x_2(t) + 2\,x_3(t)$,
$x_1(0) = 1, x_2(0) = 2, x_3(0) = 5$.

a) Enter this IVP into Maple using "*x1*" etc for the subscripted variables to avoid wasting time with subscripts. Right click and solve the system and write down the vector solution $\vec{x} = \langle x_1, x_2, x_3\rangle$ from Maple's output. Rewrite this as a linear combination of the two independent exponential functions which enter into that solution, whose coefficients are constant vectors, i.e., $\vec{x} = e^{a\,t}\,\vec{u} + e^{b\,t}\,\vec{v}$.

b) Rewrite this system of DEs **and** its initial conditions explicitly in matrix form for the vector variable $\boldsymbol{x} = \langle x_1, x_2, x_3\rangle$ as a column matrix (using the actual matrices, not their symbols), identifying the coefficient matrix A and suppressing function notation in the DE [x_1' not $x_1'(t)$].

c) For this A, using Maple (Rightclick, Eigenvalues etc, Eigenvectors) write down the eigenvalues $\lambda_1 \le \lambda_2 \le \lambda_3$ (ordered by increasing value, they are integers!) and corresponding matrix of eigenvectors $B = \langle \vec{b}_1 | \vec{b}_2 | \vec{b}_3 \rangle$ that it provides you, reordering them if necessary to order them as requested.

d) Use technology to evaluate and write down the inverse matrix B^{-1} and use Maple to evaluate the matrix product $A_B = B^{-1} A\, B$. Write down this result. Compare it to the corresponding list of eigenvalues in the same order as the eigenvectors.

e) Given that $\vec{x} = B\,\vec{y}$, if $\vec{x}(0) = \langle 1, 2, 5\rangle$, use the inverse matrix to find $\vec{y}(0)$, showing the matrix multiplication steps by hand (to prove that you actually know how to multiply a vector by a matrix, but check with Maple!).

▶ solution

① a)
$$\boxed{\begin{aligned}\langle x_1,x_2,x_3\rangle &= \langle 3e^{2t}-2e^{-t},\ 3e^{2t}-e^{-t},\ 3e^{2t}+2e^{-t}\rangle \\ &= e^{2t}\langle 3,3,3\rangle + e^{-2t}\langle -2,-1,2\rangle\end{aligned}}$$

b)
$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \underbrace{\begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

c)
$$B = \begin{array}{ccc} \lambda_1=\text{[illegible]} & \lambda_2=\text{[illegible]} & \lambda_3=2 \\ \left[\begin{matrix} -\frac{1}{2} \\ 0 \\ 1 \end{matrix}\right. & \begin{matrix} 1 \\ 1 \\ 0 \end{matrix} & \left.\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\right] \\ \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{array}$$

d)
$$B^{-1} = \begin{bmatrix} -2 & 2 & 0 \\ -2 & 3 & -1 \\ 2 & -2 & 1 \end{bmatrix}$$

d) continued
$$B^{-1}AB = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$
diagonal values = ordered eigenvalues

e)
$$\vec{x}(0) = B\,\vec{y}(0) \rightarrow \vec{y}(0) = B^{-1}\vec{x}(0) = \begin{bmatrix} -2 & 2 & 0 \\ -2 & 3 & -1 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -2(1)+2(2)+0(5) \\ -2(1)+3(2)-1(5) \\ 2(1)-2(2)+1(5) \end{bmatrix} = \begin{bmatrix} -2+4 \\ -2+6-5 \\ 2-4+5 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \vec{y}(0)}$$