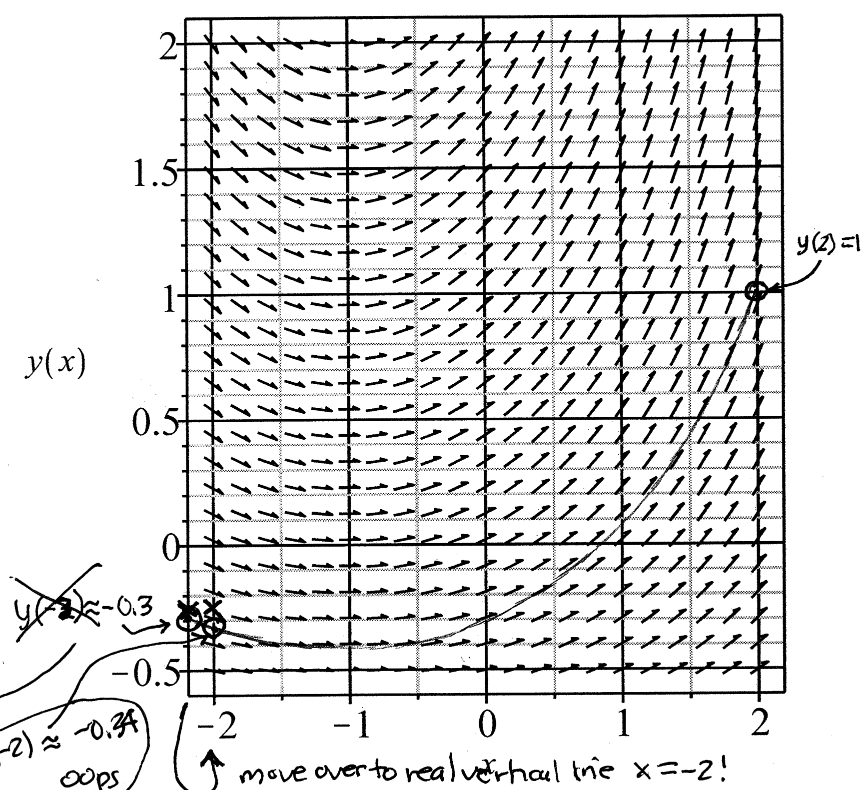
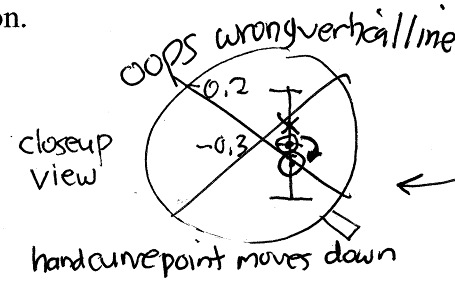
MAT2705-01/02 16F Quiz 3 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $4y' = 1 + x + y + xy$, $y(2) = 1$.
a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition, annotated by an arrow to its location from the label $y(2) = 1$. Similarly indicate the point on this curve where $x = -2$ and estimate $y(-2)$. Don't change your curve or estimated value after you later evaluate the solution exactly.
b) Use the linear solution recipe to find the general solution of this differential equation.
c) Find the solution of this differential equation which satisfies the given initial condition.





d) Evaluate your solution at $x = -2$ numerically to 2 decimal places and mark the corresponding point on your graph with a visible × . Is this consistent with your part a) result? Explain.
e) Does your initial value problem solution agree with Maple (check with the original form of the DE!)? If equivalent, show the equivalence. If not, can you find your mistake?

▶ solution

b) $4y' = (1+x) + (1+x)y$

$4y' - (1+x)y = 1+x$

$e^{-\frac14(x+\frac{x^2}{2})}\left[y' - \frac14(1+x)y = \frac14(1+x)\right] \to \frac{d}{dx}\left(ye^{-\frac14(x+\frac{x^2}{2})}\right) = \frac14(1+x)e^{-\frac14(x+\frac{x^2}{2})}$

$e^{\int -\frac14(1+x)\,dx} = e^{-\frac14(x+\frac{x^2}{2})}$

$ye^{-\frac14(x+\frac{x^2}{2})} = \int \frac14(1+x)\, e^{\overbrace{-\frac14(x+\frac{x^2}{2})}^{u}}\, \underbrace{dx}_{-du} = -\int e^u\,du$

$= -e^{-\frac14(x+\frac{x^2}{2})} + C$

$y = e^{\frac14(x+\frac{x^2}{2})}\left[-e^{-\frac14(x+\frac{x^2}{2})} + C\right]$

$\boxed{y = -1 + Ce^{\frac14(x+\frac{x^2}{2})}} = -1 + Ce^{\frac{x}{8}(2+x)}$

alternative: $\int \frac14 \underbrace{(1+x)}_{u} \underbrace{dx}_{du} = -\frac14\int u\,du = -\frac18 u^2 = -\frac18(x+1)^2$ simpler!

c) $1 = y(2) = -1 + Ce^{\frac14(2+\frac42)}$

$= -1 + Ce^1$

$2 = Ce$, $C = 2e^{-1}$

$$\boxed{\begin{aligned} y &= -1 + 2e^{-1}e^{\frac14(x+\frac{x^2}{2})} \quad \text{ok} \\ &= -1 + 2e^{\frac14(x+\frac{x^2}{2})-1} \quad \text{simpler} \end{aligned}}$$

d) $y(-2) = -1 + 2e^{0-1}$

$= -1 + 2e^{-1} \approx -0.2642$

$\approx \boxed{-0.26}$

pretty close to my estimate of -0.34

e) Maple: $y(x) = -1 + \frac{2e^{\frac18 x(2+x)}}{e}$

> combine(%) puts e into exponential