MAT2705-01/02 16f Quiz 1 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $x\frac{dy}{dx} + 3y = 2x^5$, *gen soln* : $y = \frac{1}{4}x^5 + Cx^{-3}$

a) Verify that this y satisfies the given differential equation.

b) Find the solution which satisfies the initial condition $y(1) = 2$ $\quad y(1)=2!$

Organize your work as though you were playing professor.

2. Choose appropriately named variables and write a differential equation that models the situation:

"A spherical balloon is being blown up inside a spherical cavity of volume V_S filled with liquid in such a way that the time rate of change of the balloon's volume is inversely proportional to the balloon's volume and directly proportional to the difference between its volume and V_S." [Translation: the time rate of change is proportional to the the difference in volume divided by the balloon volume itself.]

If the balloon is being blown up as stated, what should the sign of your proportionality constant be?

▶ solution

replace y everywhere in DE & simplify both sides:

① a) $x\frac{d}{dx}\left(\frac{1}{4}x^5 + Cx^{-3}\right) + 3\left(\frac{1}{4}x^5 + Cx^{-3}\right) = 2x^5$

$x\left(\frac{5}{4}x^4 - 3Cx^{-4}\right) + \frac{3}{4}x^5 + 3Cx^{-3} = 2x^5$

$\underbrace{\left(\frac{5}{4} + \frac{3}{4}\right)}_{2} x^4 - \cancel{3Cx^{-3}} + \cancel{3Cx^{-3}} \to 0 = 2x^5$

$2x^5 = 2x^5$ ✓

b) $2 = y(1) = \frac{1}{4}(1) + C(1) = C + \frac{1}{4} \to C = 2 - \frac{1}{4} = \frac{7}{4}$

$$\boxed{y = \frac{1}{4}x^5 + \frac{7}{4}x^{-3}}$$

② $\frac{dV}{dt} \propto \frac{V_s - V}{V}$ $\qquad$ Note $0 < V \le V_s$ (balloon is <u>inside</u> cavity)

↑ $V \neq 0$ division by 0!

$$\boxed{\frac{dV}{dt} = k\underbrace{\frac{V_s - V}{V}}_{>0 \text{ if } V < V_s}}$$

↓

$k > 0$ volume increases (balloon is being "blown up")

if you instead wrote $\frac{dV}{dt} \propto \frac{V - V_s}{V} \to \frac{dV}{dt} = k\left(\frac{V - V_s}{V}\right)$ then $k < 0$.