MAT2705-01/02 16F Test 1 Print Name (Last, First) Jantzen, bob

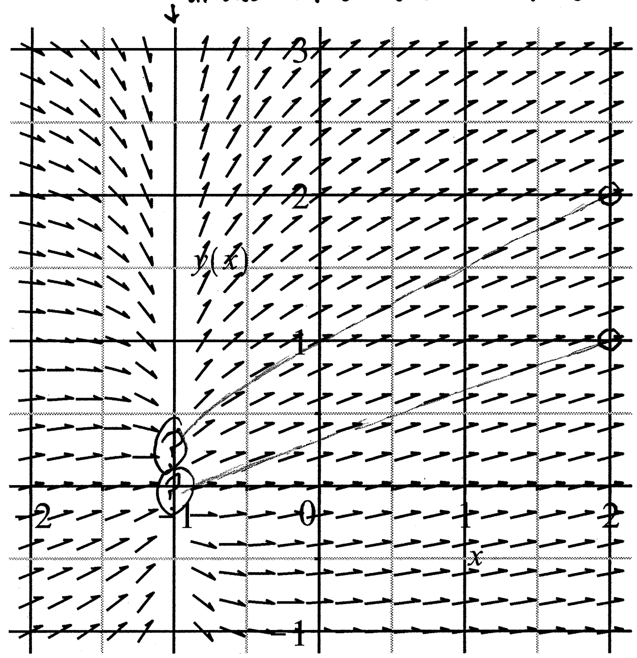
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). [Recall you need $y'(t), y(t)$ instead of y', y in your differential equation for an unknown variable y for Maple to interpret the prime as a t derivative.]

$x=-1$ must be finite; must be zero if $x=-1$: all soln curves pass through $(-1,0)$.

1. $\underbrace{4(1+x)}_{0} \underbrace{\frac{dy}{dx}}_{\text{must be finite}} = \underbrace{1+x}_{0}+y$

arrows vertical here: $\underbrace{4(1+x)}_{0} = (\underbrace{1+x}_{0}+y)\frac{dx}{dy} \to 0$

a) Locate the points $(2, 1)$ and $(2, 2)$ on the direction field plot and carefully draw in the solution curves through those points.
b) Put the DE into standard form for a linear first order DE.
c) Find the general solution.
d) Find the solutions of the two initial value problems with the initial points $(2, 1)$ (=sol1) and $(2, 2)$ (= sol2).
e) Evaluate each of these solutions at $x=-1$. Evaluate your general solution there. What can you say about the solution curves for all initial data points with $x>-1$?
f) Check your solution to the initial value problem with $y(2)=2$ following bob's method of checking any equation: replace the unknown everywhere in the equation and simplify both sides independently.



$x=-1$ is a solution for y as ind. var.

not enough info from grid to follow curves to $x=-1$.

2. $\frac{dP}{dt} = P(k+b\cos(2\pi t)),\ P(0)=P_0$

Birth and death rates of animal populations typically oscillate periodically with the passage of seasons, describable by the above simple model. Note that the growth rate function $r(t)=k+b\cos(2\pi t)$ varies periodically about its average value k, indeed with $b=0$ one has purely exponential behavior $P_{\exp}=P_0 e^{kt}$.
a) Solve this initial value problem by the separable DE method.
b) Suppose $P_0=100$, $k=0.03$, $\frac{dP}{dt}(0)=9$. Use the DE at $t=0$ to find the value of b. What is the maximum percent that the population exceeds the average exponential growth during any given period (i.e., examine $P/P_{\exp}$).
c) What is the period of the oscillation? How many periods fit into one decay time of the exponential factor?

▶ solution

▼ pledge [Be sure to sign and date the pledge before handing in this test.]

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

MAT 2705-01/02 16F Test 1 Answers

1 b) $\left[4(1+x)\frac{dy}{dx} = 1+x+y\right]\frac{1}{4(1+x)}$

$$\frac{dy}{dx} = \frac{1}{4(1+x)}y + \frac14$$

$$\boxed{\frac{dy}{dx} - \underbrace{\frac{1}{4(1+x)}}\,y = \frac14}$$ standard linear form

c) $$e^{\int -\frac{1}{4(1+x)}dx} = e^{-\frac14 \ln|1+x|} = |1+x|^{-1/4}$$

$$|1+x|^{-1/4}\left(\frac{dy}{dx} - \frac{y}{4(1+x)}\right) = \frac14 |1+x|^{-1/4}$$

$$\frac{d}{dx}\left(y\,|1+x|^{-1/4}\right) = \frac14|1+x|^{-1/4}$$

$$y(1+x)^{-1/4} = \int \frac14(1+x)^{-1/4}dx,\quad x+1>0$$

$$= \frac14\,\frac{(1+x)^{3/4}}{3/4} + C$$

$$y = (1+x)^{1/4}\left(\frac13(1+x)^{3/4} + C\right)$$

$$\boxed{y = \frac13(1+x) + C(1+x)^{1/4}}$$ if $1+x>0$

d) (2,1): $1 = \frac13(1+2) + C(1+2)^{1/4}$

$0 = C\,3^{1/4}$; $C = 0$

$$\boxed{y = \frac13(1+x) + \cancel{\frac{(1+x)^{1/4}}{3\cdot 2^{1/4}}}}$$

(2,2): $2 = \underbrace{\frac13(2+1)}_{1} + C(2+1)^{1/4}$

$1 = C(3)^{1/4}$, $C = 3^{-1/4}$

$$\boxed{y = \frac13(1+x) + \frac{(1+x)^{1/4}}{3^{1/4}}}$$

e) sol1: $y(-1) = \frac13(1-1) + \cancel{\frac{(1-1)^{1/4}}{3\cdot 2^{1/4}}} = 0$

sol2: $y(-1) = \frac13(1-1) + \frac{(1-1)^{1/4}}{3^{1/4}} = 0$

gen sol ($x>-1$): $y(-1) = 0$! they all must go through ~~origin~~ pt $(-1,0)$. [not clear from graph]

f) $$4(x+1)\frac{d}{dx}\left(\frac13(1+x) + \frac{(1+x)^{1/4}}{3^{1/4}}\right) = 1+x+\frac13(1+x) + \frac{(1+x)^{1/4}}{3^{1/4}}$$

$$4(x+1)\left(\frac13 + \frac{1}{4\cdot 3^{1/4}(1+x)^{3/4}}\right) = \frac43(1+x) + \frac{(1+x)^{1/4}}{3^{1/4}}$$

$$\frac43(x+1) + \frac{1}{3^{1/4}}(x+1)^{1/4} = \frac43(x+1) + \frac{(x+1)^{1/4}}{3^{1/4}} \checkmark$$

check the D.E.! not the initial condition

2 a) $$\frac{dP}{dt} = P(k + b\cos(2\pi t)),\quad P>0$$

$$\int \frac{dP}{P} = \int (k + b\cos 2\pi t)\,dt$$

$$\ln P = kt + \frac{b}{2\pi}\sin 2\pi t + C_1$$

$$P = e^{kt + \frac{b}{2\pi}\sin 2\pi t + C_1}$$

$$= e^{C_1}e^{kt}e^{\frac{b}{2\pi}\sin 2\pi t}$$

$$= Ce^{kt}e^{\frac{b}{2\pi}\sin 2\pi t}$$

$$P_0 = P(0) = Ce^0e^0 = C \to C = P_0$$

$$\boxed{P = P_0e^{kt}e^{\frac{b}{2\pi}\sin 2\pi t}}$$

b) $$\left.\frac{dP}{dt}\right|_{t=0} = P_0(k+b)$$

$9 = 100(.03 + b)$

$= 3 + 100b$

$6 = 100b$, $\boxed{b = .06}$

decimal notation appropriate from problem setup.

$$\frac{P}{P_{exp}} = \frac{P}{P_0e^{kt}} = e^{\frac{b}{2\pi}\sin 2\pi t}$$ ← max when $2\pi t = \frac{\pi}{2}$, $\sin 2\pi t = 1$

$e^{\frac{b}{2\pi}}$ max fractional amount bigger

$= e^{\frac{.06}{2\pi}} \approx 1.009595 \approx 1.0096$

about 0.96% larger

c) period of sin x: 

$\sin 2\pi t$, $2\pi t = 2\pi \to t = 1 =$ period "T" (1 year!)

$k = .03$, $\tau = \frac1k = \frac{1}{.03} \approx 33.3$ (yrs!)

$\frac{T}{\tau} = \frac{1}{33\frac13} = .03$

$\frac{\tau}{T} = \frac{33\frac13}{1}$

like music records of last century

$\boxed{33\tfrac13 \text{ periods fit into one characteristic time}}$

not one person asked what the "decay" time is. clearly it should have been the "growth" time a mistake in wording — But there is only one characteristic time associated with exponential behavior; τ the "characteristic" time which characterizes the timescale of the exponential behavior