MAT2705-01/02 16f Quiz 0 Print Name (Last, First) _____, _____|____
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y''+y=3\cos(2x) : sol : y=\cos(x)-\cos(2x)$
Verify that this y satisfies the given differential equation.
Organize your work as though you were playing professor.

2. Express this proportionality as an equation:
"The gravitational force F is inversely proportional to the square of the distance r from the point mass m but directly proportional to that mass." Does this sentence contain enough information to determine the sign of the constant of proportionality?

▶ solution

substitute for y everywhere in both sides of DE, then simplify each side

(1) $y = \cos x - \cos 2x$
$y' = -\sin x + 2\sin 2x$
$y'' = -\cos x + 4\cos 2x$

$y'' + y = 3\cos 2x$

$(-\cos x + 4\cos 2x) + (\cos x - \cos 2x) = 3\cos 2x$

$3\cos 2x = 3\cos 2x$ ✓

(2) "as an equation", i.e., one equation, not two (read carefully the words of each problem)

$\boxed{F = k\frac{m}{r^2}}$

The sign depends on whether we are talking about the radial projection or magnitude of the force vector, which points inwards towards the point mass, so no, there is insufficient information to determine the sign of k.

Note: F is inversely proportional to r^2 : $F = \frac{k_1}{r^2}$, k_1 independent of r

F is directly proportional to m : $F = k_2 m$, k_2 independent of m

different "constants" of proportionality

but we need a single expression for F

$\left[k_1 = km,\ k_2 = \frac{k}{r^2}\right]$

distinct, you cannot use the same symbol to stand for different quantities in the same problem

Highschool? Who has never seen Newton's law $F = \frac{Gm_1m_2}{r^2}$?

or the electric field equivalent "Coulomb's law" $F = k_e\frac{q_1q_2}{r^2}$ (wikipedia) ?

These two simple laws rule our lives!