MAT2705-02/05 15s Quiz 1 Print Name (Last, First) _____, _____|____
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y' = 2 - y$: *gen sol* : $y(x) = C e^{-x} + 2$
 a) Verify that this y satisfies the given differential equation.
 b) Find the solution which satisfies the initial condition $y(0) = -1$.
Organize your work as though you were playing professor.

2. Choose appropriately named variables and write a differential equation that models the situation:
"A metal company's production measured by the number of units produced is increasing at a rate proportional to the product of the number of units produced and the time in years starting from the year 2000." What is the sign of the constant of proportionality you introduced? Explain.

▶ solution

① a) $y' = 2 - y \leftarrow y = Ce^{-x} + 2$

$\frac{d}{dx}(Ce^{-x} + 2) = 2 - (Ce^{-x} + 2)$

$Ce^{-x}(-1) + 0 = -Ce^{-x}$

$-Ce^{-x} = -Ce^{-x}$ ✓

b) $y(0) = -1 \leftrightarrow x = 0, y = -1$

$y = Ce^{-x} + 2$

$-1 = Ce^{-0} + 2 = C + 2 \rightarrow C = -1 - 2 = -3$

$\boxed{y = 2 - 3e^{-x}}$

② Let N = # units produced

$\frac{dN}{dt} \propto N(t - 2000)$

$\boxed{\frac{dN}{dt} = kN(t - 2000)}$ if t = calendar year

"time in years starting from the year 2000" can be interpreted in two ways depending on the zero of the time t.

when there are ambiguities it is crucial to describe what a variable actually measures.

OR: $\boxed{\frac{dN}{dt} = kNt}$ if t = # years from 2000

note $\boxed{k > 0}$ since population is increasing so $\frac{dN}{dt}$ must be positive.