MAT1505-03/04 15F Quiz 8   Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $a_n = \sqrt{\dfrac{1+4n^2}{1+n^2}}$.

a) Show the evaluation of $\lim_{n\to\infty} a_n$.

b) Evaluate the decimal values of the first 10 terms on your favorite tech device, but don't bother listing them on this sheet. Do these seem to confirm part a)? If so, at which term in the sequence (give n and a_n to 5 decimal places) does the term get within 1 percent of the limiting value? If not, reconsider part a).

2. The concentration of drug in the blood immediately following1.5mg/mL injections every 12 hours, during which that concentration decreases by 90 percent, is described by the partial sums C_n of a geometric series:

$C_n = \sum_{k=1}^{n} 1.5 \cdot 0.1^{k-1}$. Let $C = \sum_{k=1}^{\infty} 1.5 \cdot 0.1^{k-1}$ be the limiting concentration.

a) Write down the numerical values of the first four partial sums.

b) What are the first term and ratio of this geometric series?

c) What is the numerical value of C ? Can you guess what the exact value is?

Optional: What is the limiting concentration just before injection? Once the limiting concentration C is "effectively reached" just after each injection, the concentration during one 12 hour cycle measured in units of 12 hours from its start is given by $c = C\,e^{-\ln(10)\,x}$. What is the average value of the concentration during one such cycle?

▶ solution

① a) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{\dfrac{1+4n^2}{1+n^2}} = \lim_{n\to\infty} \sqrt{\dfrac{(1+4n^2)/n^2}{(1+n^2)/n^2}} = \lim_{n\to\infty} \sqrt{\dfrac{\cancel{\frac{1}{n^2}}^{\to 0}+4}{\cancel{\frac{1}{n^2}}^{\to 0}+1}} = \sqrt{\dfrac{4}{1}} = \boxed{2}$

$\left[\text{OR} = \lim_{n\to\infty} \sqrt{\dfrac{\cancel{1}+4n^2}{\cancel{1}+n^2}} = \lim_{n\to\infty} \sqrt{\dfrac{4n^2}{n^2}} = \lim_{n\to\infty} \sqrt{4} = 2 \right]$

can neglect for large n

b) $a_{10} \approx 1.99256$ they seem to be approaching 2 numerically.
1% of 2 is .02 so 1.98 is 99% of the limiting value, $\boxed{a_7 \approx 1.98494}$ is the first term to cross over that line.

② a) $\{C_n\}_{n=1}^{4} = \{1.5, 1.65, 1.665, 1.6665\}$

b) $C = \sum_{k=1}^{\infty} 1.5 \cdot 0.1^{k-1} \to a = \boxed{1.5 = a}$, $\boxed{r = 0.1}$

c) $C = \dfrac{a}{1-r} = \dfrac{1.5}{1-0.1} = \dfrac{1.5}{.9} \approx \boxed{1.6667}$
↓
6 repeating

$\left(= \dfrac{3/2}{9/10} = \dfrac{5}{3}! \right)$
("exact arithmetic")

looks like: $\boxed{1\,2/3}$

optional: 1.5 less: $C - 1.5 \approx 1.6667 - 1.5 \approx \boxed{0.1667}$

$\left(\frac{5}{3} - \frac{3}{2} = \frac{10-9}{6} = \frac{1}{6}! \right)$

$C_{avg} = \frac{1}{1}\int_0^1 C\,e^{-(\ln 10)x}\,dx = C\,\dfrac{e^{-\ln 10\, x}}{-\ln 10}\Big|_0^1 = \dfrac{C}{\ln 10}\big(1 - \underbrace{e^{-\ln 10}}_{\frac{1}{10}}\big) \approx \dfrac{.9C}{\ln 10} \approx \boxed{0.65144}$ see plot in Maple worksheet

$\left(= \dfrac{3}{2\ln 10} \right)$