MAT1505-03/04 15F Quiz 7 Print Name (Last, First) ________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approxim ations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. $f(x) = \dfrac{x^4}{16} + \dfrac{1}{2x^2}$ on the interval $1 \le x \le 2$.

a) Make a rough plot of the graph of this function only on this interval, and include the secant line connecting its endpoints, labeling the coordinates of those endpoints.

b) Evaluate the length of the secant line numerically.

c) Write down a simplified definite integral for the arclength function $s(x)$ for this function starting from the reference point on its graph at $x = 1$, for $x > 0$. Note that the integrand radical contains a perfect square which simplifies the integrand, allowing you to integrate this easily by hand. In fact show that this integrand simplifies to:

$\dfrac{1}{4}x^3 + \dfrac{1}{x^3}$.

d) Now evaluate this simplified definite integral by hand to obtain the arclength function.

e) Use it to evaluate the (exact!) arclength of this segment of the graph on this interval. Compare it to your length of the secant line---is it reasonable?

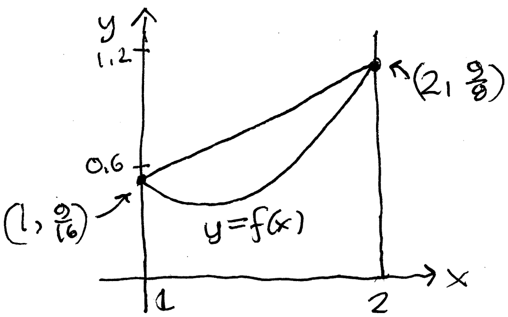
▶ solution

① a) $f(x) = \dfrac{x^4}{16} + \dfrac{1}{2x^2} = \dfrac{1}{16}x^4 + \dfrac{1}{2}x^{-2}$

$f'(x) = \dfrac{4}{16}x^3 + \dfrac{1}{2}(-2)x^{-3} = \dfrac{1}{4}x^3 - \dfrac{1}{x^3}$

$f(1) = \dfrac{1}{16} + \dfrac{1}{2} = \dfrac{9}{16}$

$f(2) = \dfrac{16}{16} + \dfrac{1}{8} = \dfrac{9}{8}$



b) $L_{sec} = \sqrt{(2-1)^2 + \left(\frac{9}{8} - \frac{9}{16}\right)^2}$

$\approx \boxed{1.147}$

c) $s(x) = \displaystyle\int_1^{x} \sqrt{1 + f'(t)^2}\, dt$

$= \displaystyle\int_1^x \sqrt{1 + \left(\tfrac{1}{4}t^3 - \tfrac{1}{t^3}\right)^2}\, dt$

$\left(\tfrac{1}{4}t^3\right)^2 - 2\left(\tfrac{1}{4}t^3\right)\left(\tfrac{1}{t^3}\right) + \left(\tfrac{1}{t^3}\right)^2$

$+$

$\left(\tfrac{1}{4}t^3 + \tfrac{1}{t^3}\right)^2$

adding 1 only changes sign of cross-term

remark..
Since x is the upper limit, the preferred math notation is to change the "dummy variable" in the definite integral to avoid confusion. however, if you leave the integration variable as x, it won't lead to any error once you subtract the values of the antiderivative at the upper & lower limits.

② continued

$s(x) = \displaystyle\int_1^x \tfrac{1}{4}t^3 + t^{-3}\, dt$

$= \dfrac{1}{16}t^4 + \dfrac{t^{-2}}{-2} \Big|_1^x$

$= \dfrac{x^4}{16} - \dfrac{1}{2x^2} - \left(\dfrac{1}{16} - \dfrac{1}{2}\right)$

$= \boxed{\dfrac{x^4}{16} - \dfrac{1}{2x^2} + \dfrac{7}{16}}$

d) $s(2) = \dfrac{16}{16} - \dfrac{1}{8} + \dfrac{7}{16} = \dfrac{16 - 2 + 7}{16}$

$= \boxed{\dfrac{21}{16}} \approx 1.3125$:

compared to 1.147 it is a bit larger as expected so yes it is reasonable.