MAT 2505-03/04 15F Quiz 7 comments

FUNCTION NOTATION:

many of you write:

"dummy variable" in definite integral with constant limits — doesn't matter what symbol you use because the evaluated integral does not depend on it

$$S(x) = \int_1^2 \sqrt{1+f'(x)^2}\,dx \quad \left(= \int_1^2 \sqrt{1+f'(t)^2}\,dt = \int_1^2 \sqrt{1+f'(u)^2}\,du \right)$$

suppose an antiderivative is $F(x)$

then RHS $= F(x)\Big|_1^2 = F(2) - F(1)$ = evaluated number, NOT a function of x

this notation implies that when the RHS is fully evaluated, ~~it no longer de~~ it is an expression involving the variable x

indeed this stands for the variable arclength of the graph of f from $x=1$ to a general value of x here:

$$S(x) = \int_1^{x} \sqrt{1+f'(t)^2}\,dt \qquad = F(x) - F(1) \text{ for a general antiderivative}$$

dummy variable

dependence on x through upper limit of integral

BINOMIAL THEOREM:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{"binomial coefficient"}$$

$$(a+b)^0 = 1$$
$$(a+b)^1 = a + b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$\ldots$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Pascal's triangle: sum of two adjacent coefficients equals coefficient below!

why am I pointing this out?

many of you did: $(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2$

instead of: $(a+b)^2 = a^2 + \underbrace{2ab}_{\text{cross-term}} + b^2$

squared terms

yes, they both give the same final result but not using the "squaring formula" reflects a lack of mathematical thinking.

in the end these exercises serve only one purpose — to help train your mind to think mathematically to understand how you can use mathematics to quantitatively understand how the world around you works.