MAT1505-03/04 15F Quiz 4 Print Name (Last, First) ________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. The curve $y = h\left(\dfrac{x^2}{R^2} - 1\right)$ over the interval $0 \le x \le R$ is rotated around the y-axis to form a parabolic tank with its rim on the x-axis.

a) Let $-H > 0$ designate the average value of y over this interval. Write down a simplified definite integral for H and then evaluate it.

b) Make a diagram of this curve over this interval and include a horizontal line for this average value. Does the rectangle it makes appear to have the same area as that above the curve? Explain.

c) Let V denote the volume contained in this parabolic tank, and indicate a typical horizontal cross-section of the tank needed to evaluate this volume, labeling its endpoints appropriately to justify your limits of integration and indicating how you obtained your integrand. Write down a simplified definite integral for V and then evaluate it.

d) The tank is filled with a liquid whose weight density (lbs per cubic ft) is 1 and all of it is pumped through a pipe along the y-axis to a height $y = h$. Evaluate the work W done to accomplish this. [No units needed.]

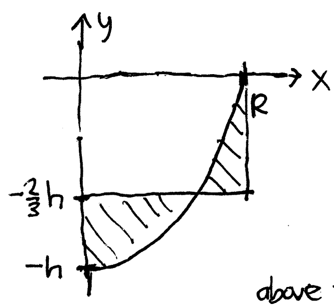
e) Show that $W = 2\,H\,V$.

▶ solution

① a) $-H = \boxed{\dfrac{\int_0^R h\left(\frac{x^2}{R^2}-1\right)dx}{R}} = \dfrac{h}{R}\left(\dfrac{x^3}{3R^2}-x\right)\Big|_0^R$

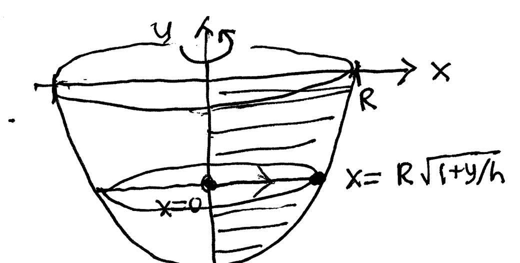
$= \dfrac{h}{R}\left(\dfrac{R^3}{3R^2}-R\right) = \boxed{-\dfrac{2}{3}h}$

b)



well, to tell the truth the two shaded areas don't seem to be equal, which is necessary if the rectangle area is to equal the area above the curve.

c)



$y = h\left(\frac{x^2}{R^2}-1\right) \xrightarrow[\text{for } x]{\text{solve}} \frac{y}{h} = \frac{x^2}{R^2} - 1$

$\frac{x^2}{R^2} = \frac{y}{h} + 1 \rightarrow x^2 = R^2(1+y/h)$

$x = \pm R\sqrt{1+y/h}$

$A(y) = \pi x(y)^2 = \pi R^2(1+y/h)$

$V = \int_{-h}^0 A(y)\,dy = \boxed{\int_{-h}^0 \pi R^2(1+y/h)\,dy}$

c) continued

$V = \pi R^2\left(y + \frac{y^2}{2h}\right)\Big|_{-h}^0$

$= -\pi R^2\left(-h + \frac{h^2}{2h}\right) = \pi R^2\left(h - \frac{1}{2}h\right)$

$= \boxed{\frac{1}{2}\pi R^2 h}$

d) $\Delta V = \pi x(y)^2 \Delta y$

$1\Delta V = \pi R^2\left(1+\frac{y}{h}\right)\Delta y$ = g-force on slab Δy

$\Delta W = 1\Delta V\,\underbrace{(h-y)}_{\text{distance } \Delta y \text{ slab lifted.}}$

$\downarrow$

$W = \boxed{\int_{-h}^0 \pi R^2\left(1+\frac{y}{h}\right)(h-y)\,dy}$

$= \int_{-h}^0 \pi R^2\left(h + \cancel{y} - \cancel{y} - \frac{y^2}{h}\right)dy$

$= \pi R^2\left(hy - \frac{y^3}{3h}\right)\Big|_{-h}^0 = -\pi R^2\left(-h^2 + \frac{h^3}{3h}\right)$

$= \boxed{\frac{2}{3}\pi R^2 h}$

d) $2HV = 2\left(\frac{2}{3}h\right)\left(\frac{1}{2}\pi R^2 h\right)$

$= \frac{2}{3}\pi R^2 h = W$ ✓