MAT1505-03/04 15F Quiz 3 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. The region R is bounded by the curves $y = 2x$ and $y = 2\sqrt{x}$ over the interval $0 \le x \le 1$.

a) Make a diagram of this region in the first quadrant, labeling axes and curves appropriately.

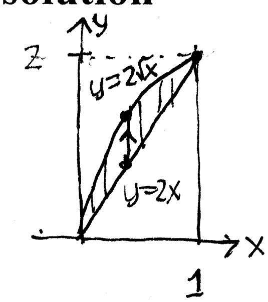
b) Rotate this region around the axis $y = 0$. Write down an integral V_1 (with simplified integrand) representing the volume of the resulting solid of revolution, and support your expression by a new diagram shading the region R with appropriate linear cross-sections, one of which is labeled appropriately at its bullet endpoints corresponding to the limits of integration. Show also the reflection of the region across the axis and indicate any other lengths in the diagram relevant to the integrand of your integral.

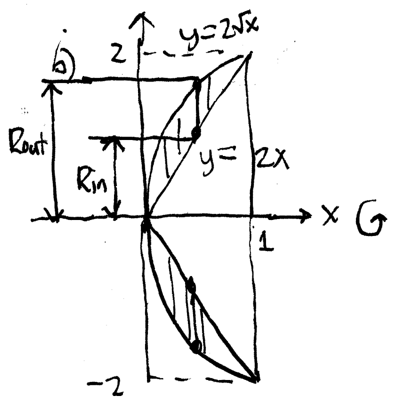
c) Rotate this region around the axis $x = 1$. Repeat the instructions of b) for the corresponding volume V_2.

d) Evaluate V_1 and V_2 exactly with technology. [Check: $\frac{V_1}{V_2} = \frac{5}{3}$.]

▶ solution

1 a)





$R_{out} = 2\sqrt{x}$

$R_{in} = 2x$

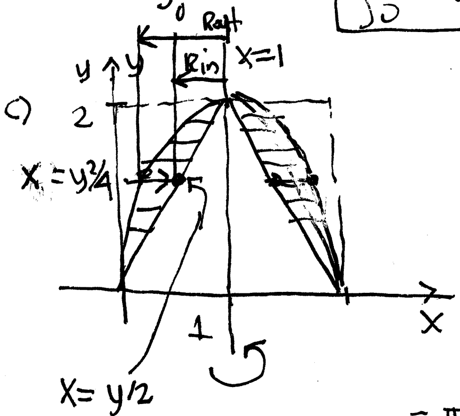
$A(x) = \pi(R_{out}^2 - R_{in}^2)$

$= \pi[(2\sqrt{x})^2 - (2x)^2]$

$= \pi[4x - 4x^2]$

$= 4\pi(x - x^2)$

$V_1 = \int_0^1 A(x)\,dx = \boxed{\int_0^1 4\pi(x - x^2)\,dx}$



$y = 2x \rightarrow x = y/2$

$y = 2\sqrt{x} \rightarrow x = y^2/4$

$R_{out} = 1 - y^2/4$

$R_{in} = 1 - y/2$

$A(y) = \pi(R_{out}^2 - R_{in}^2)$

$= \pi[(1 - \frac{y^2}{4})^2 - (1 - \frac{y}{2})^2]$

$= \pi[\cancel{1} - \frac{y^2}{2} + \frac{y^4}{16} - (\cancel{1} - y + \frac{y^2}{4})]$

c) continued

$A(y) = \pi(\frac{y^4}{16} - \frac{3}{4}y^2 + y)$

$V_2 = \int_0^2 A(y)\,dy = \boxed{\int_0^2 \pi(\frac{y^4}{16} - \frac{3}{4}y^2 + y)\,dy}$

d) $\boxed{V_1 = \frac{2\pi}{3},\ V_2 = \frac{2\pi}{5}}$

note $\frac{V1}{V2} = \frac{2\pi/3}{2\pi/5} = 5/3$ ✓