Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. Sketch the region between $y = 4 - x^2$ and $y = 0$ and find its area exactly and numerically to 5 significant digits. [In your sketch, be sure to label the axes and appropriate tickmarks, and the curves you plot.]

2. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes. Complete your solution with an English sentence stating the result of the word problem. [Can you think of why the time range domain for the formula is chosen? Explain.]
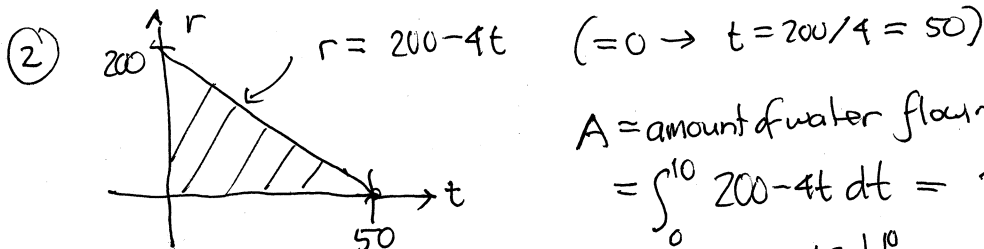
► **solution**

① $y = 4 - x^2 = 0 \rightarrow x^2 = 4$
$\qquad\qquad\qquad\qquad x = \pm 2$
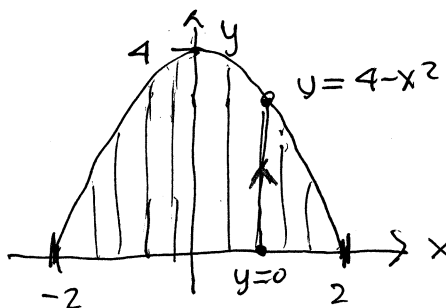
area between curves $y = 4 - x^2$ and $y = 0$

(shading conveys region of area to be found)

$y = 4 - x^2$

$y = 0$

$-2 \qquad 2$

Area $= \int_{-2}^{2} 4 - x^2 \, dx$

$\quad = 4x - x^3/3 \Big|_{-2}^{2} = \left(4(2) - \frac{8}{3}\right) - \left(4(-2) - (-8/3)\right) = 2\left(8 - \frac{8}{3}\right)$

$\quad = 16\left(1 - \frac{1}{3}\right) = 16\left(\frac{2}{3}\right) = \boxed{\frac{32}{3}} = 10\,2/3 \approx \boxed{10.667}$

② $r = 200 - 4t \qquad (= 0 \rightarrow t = 200/4 = 50)$

200

50

$A =$ amount of water flowing out in first 10 minutes

$\quad = \int_{0}^{10} 200 - 4t \, dt = 200t - 4\left(\frac{t^2}{2}\right)\Big|_{0}^{10}$

$\quad = 200t - 2t^2 \Big|_{0}^{10} = 200(10) - 2(10)^2$

$\qquad\qquad\qquad = 200(10 - 1) = 200(9)$

$\qquad\qquad\qquad = 1800$

even bob makes dumb mental errors!

$\boxed{1800 \text{ liters of water flows out of the tank in the first 10 minutes.}}$

$\boxed{\text{After 50 minutes, all the water has flowed out of the tank so the flow rate becomes zero & stays at zero.}}$

[Actually the problem does not imply the tank is empty, only that the flow of water out of the tank stops at $t = 50$.]

NOTES
① $\frac{d}{dx} f(x)$ means "d→differentiate" "/dx→with respect to x"   two part notation

$\int f(x)\,dx$ means "$\int$→integrate" $f(x)$ "dx→with respect to x"   also two part notation

② $y = f(x)$ in 2d plots label only pos the axes by variable and only put arrowheads at positive ends of axes. label curves by their equations. label intercepts, tickmarks.

③ Use proper math notation including putting an equal sign between equal expressions, use an arrow for a nonequal expression that follows another expression.