MAT2500-03/04 14S Quiz 8 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Consider the integral $\displaystyle\int_{-1}^{1}\int_{2-\sqrt{-y^2+1}}^{2+\sqrt{-y^2+1}} x \, dx \, dy$.

a) Evaluate this iterated integral exactly (no decimals) with technology.

b) Show that the two limits of integration of the inner integral correspond to the bounding curve equation $(x-2)^2 = 1 - y^2$ or $(x-2)^2 + y^2 = 1$. Draw this curve and a typical labeled cross-section of the region of the plane it contains representing the inner integal, and shade in the whole region using parallel cross-sectional lines. ["Labeled" means giving the start and stop values of the integration variable, attached to the bullet endpoints, with a central arrowhead to indicate the direction of integration.]

c) Make a new diagram corresponding to the reversed order of integration, labeling a typical cross-section appropriately.

d) Write down the corresponding new iteration of this integral.

e) Use Maple to evaluate the new iterated integral. Do it agree with part a) ?

f) Evaluate the inner integral of part a) ~~the inner definite integral~~ by hand and simplify it by expanding out the resulting expression to reduce the double integral to $4\displaystyle\int_{-1}^{1}\sqrt{1-y^2}\, dy$. Recognize this integral as four times the area under a unit semicircle. (Why?) Does this explain your result for part a)?

▶ solution

a) $\displaystyle\int_{-1}^{1}\int_{2-\sqrt{1-y^2}}^{2+\sqrt{1-y^2}} x \, dx \, dy \overset{\text{Maple}}{=} \boxed{2\pi}$

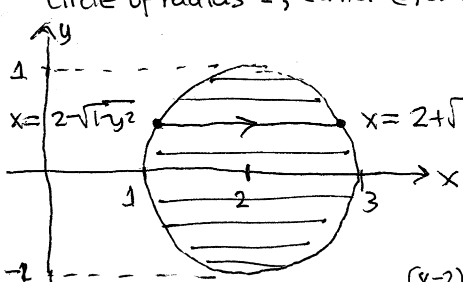
b) $x = 2-\sqrt{1-y^2} \,..\, 2+\sqrt{1-y^2}$ limits

$x - 2 = \pm\sqrt{1-y^2}$

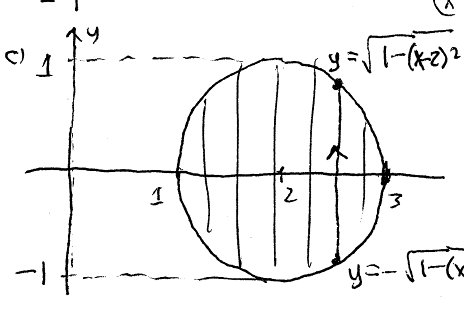
$(x-2)^2 = 1-y^2$

$(x-2)^2 + y^2 = 1$

circle of radius 1, center $(2,0)$.



c)



$(x-2)^2 + y^2 = 1$

$y^2 = 1-(x-2)^2$

$y = \pm\sqrt{1-(x-2)^2}$

d) $\displaystyle\int_{1}^{3}\int_{-\sqrt{1-(x-2)^2}}^{\sqrt{1-(x-2)^2}} x \, dy \, dx$

e) $= \boxed{2\pi}$ ✓ yes, they agree.

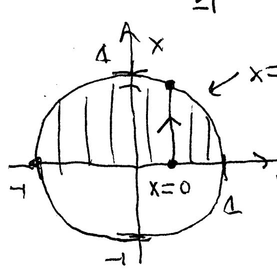
f) $\displaystyle\int_{2-\sqrt{1-y^2}}^{2+\sqrt{1-y^2}} x\,dx = \frac{x^2}{2}\Big|_{x=2-\sqrt{1-y^2}}^{x=2+\sqrt{1-y^2}}$

$= \frac{1}{2}\left[(2+\sqrt{1-y^2})^2 - (2-\sqrt{1-y^2})^2\right]$

$= \frac{1}{2}\left[\cancel{4} + 4\sqrt{1-y^2} + \cancel{(1-y^2)} - (\cancel{4} - 4\sqrt{1-y^2} + \cancel{(1-y^2)})\right]$

$= 4\sqrt{1-y^2}$

so Integral $= \displaystyle\int_{-1}^{1} 4\sqrt{1-y^2}\,dy = 4\int_{-1}^{1}\sqrt{1-y^2}\,dy$



$x=\sqrt{1-y^2}$ semicircle radius 1

area $= \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$

$x = \sqrt{1-y^2}$

$x^2 = 1-y^2$

$x^2 + y^2 = 1$

so Integral $= 4\left(\frac{\pi}{2}\right) = 2\pi$ ✓

[Recall $f \ge 0$: $\int_a^b f(x)\,dx =$ area under graph]

$\int_{-1}^{1}\sqrt{1-y^2}\,dy =$ area semicircle