MAT2500-03/04 14S Quiz 1   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1,1)$, $P_2(-1,6)$, $P_3(5,0)$ in the plane:

a) On the reverse side of this sheet (left grid), BEFORE doing this problem, draw in the three points and their position vectors, labeling each point and vector ($\overrightarrow{OP_1}$ or $\vec{r}_1$ etc), and draw in the triangle that the points determine, labeling the two sides $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$ by appropriate symbols for their difference vectors and put in arrow heads to indicate which direction your vector symbol for each side refers to.

b) First give a very rough estimate of the angle of the triangle at the vertex P_1, then evaluate it exactly (no decimals) and numerically to the nearest tenth of a degree (don't change your initial estimate!). Does your result seem consistent with estimate? Explain. [Not even I can estimate an angle better than 10 degrees without a protractor, but it should be in the same ballpark, so to speak.]

c) On the reverse side of this sheet, draw in the rectangle used to graphically project $\overrightarrow{P_1P_2}$ parallel and perpendicular to $\overrightarrow{P_1P_3}$ (use a straight edge of piece of paper to draw the lines), and draw in the vectors $\left(\overrightarrow{P_1P_2}\right)_{\parallel}$ and $\left(\overrightarrow{P_1P_2}\right)_{\perp}$ labeling the vectors and sides of the rectangle with appropriate notation for those projections. Estimate roughly the numerical components of the two vector projections.

d) Now using appropriate notation, step by step (show every step starting from the initial vector components), evaluate the vector components of the parallel and perpendicular projections that you have drawn exactly and then to 2 decimal place accuracy.

e) How do the numerical evaluations of your exact vectors compare to your graphical estimates? Do they seem consistent? Explain.

▶ solution

b) $\overrightarrow{P_1P_2} = \langle -1,6\rangle - \langle 1,1\rangle = \langle -2,5\rangle$ $|\overrightarrow{P_1P_2}| = \sqrt{4+25} = \sqrt{29}$ $\widehat{P_1P_2} = 1/\sqrt{29}\,\langle -2,5\rangle$

$\overrightarrow{P_1P_3} = \langle 5,0\rangle - \langle 1,1\rangle = \langle 4,-1\rangle$ $|\overrightarrow{P_1P_3}| = \sqrt{16+1} = \sqrt{17}$ $\widehat{P_1P_3} = 1/\sqrt{17}\,\langle 4,-1\rangle$

$$\cos\theta = \widehat{P_1P_2}\cdot\widehat{P_1P_3} = \frac{1}{\sqrt{29}}\langle -2,5\rangle\cdot\frac{1}{\sqrt{17}}\langle 4,-1\rangle = \frac{1}{\sqrt{29}\sqrt{17}}(-8-5) = -\frac{13}{\sqrt{29}\sqrt{17}}$$

$\boxed{\theta = \arccos\left(\frac{-13}{\sqrt{29}\sqrt{17}}\right) \approx 125.8^\circ}$ (compares well to guess of 130°)

d) $(\overrightarrow{P_1P_2})_{\parallel} = \underbrace{(\widehat{P_1P_2}\cdot\widehat{P_1P_3})}_{\text{scalar projection}}\widehat{P_1P_3} = \left(\langle -2,5\rangle\cdot\frac{1}{\sqrt{17}}\langle 4,-1\rangle\right)\frac{1}{\sqrt{17}}\langle 4,-1\rangle = -\frac{13}{17}\langle 4,-1\rangle = \boxed{\left\langle -\frac{52}{17},\frac{13}{17}\right\rangle}$

$(-8-5)/\sqrt{17} = -13/\sqrt{17}$

$\boxed{\approx \langle -3.06, 0.76\rangle}$

e) compare to:
$\langle -3.1, 0.75\rangle$
pretty good!

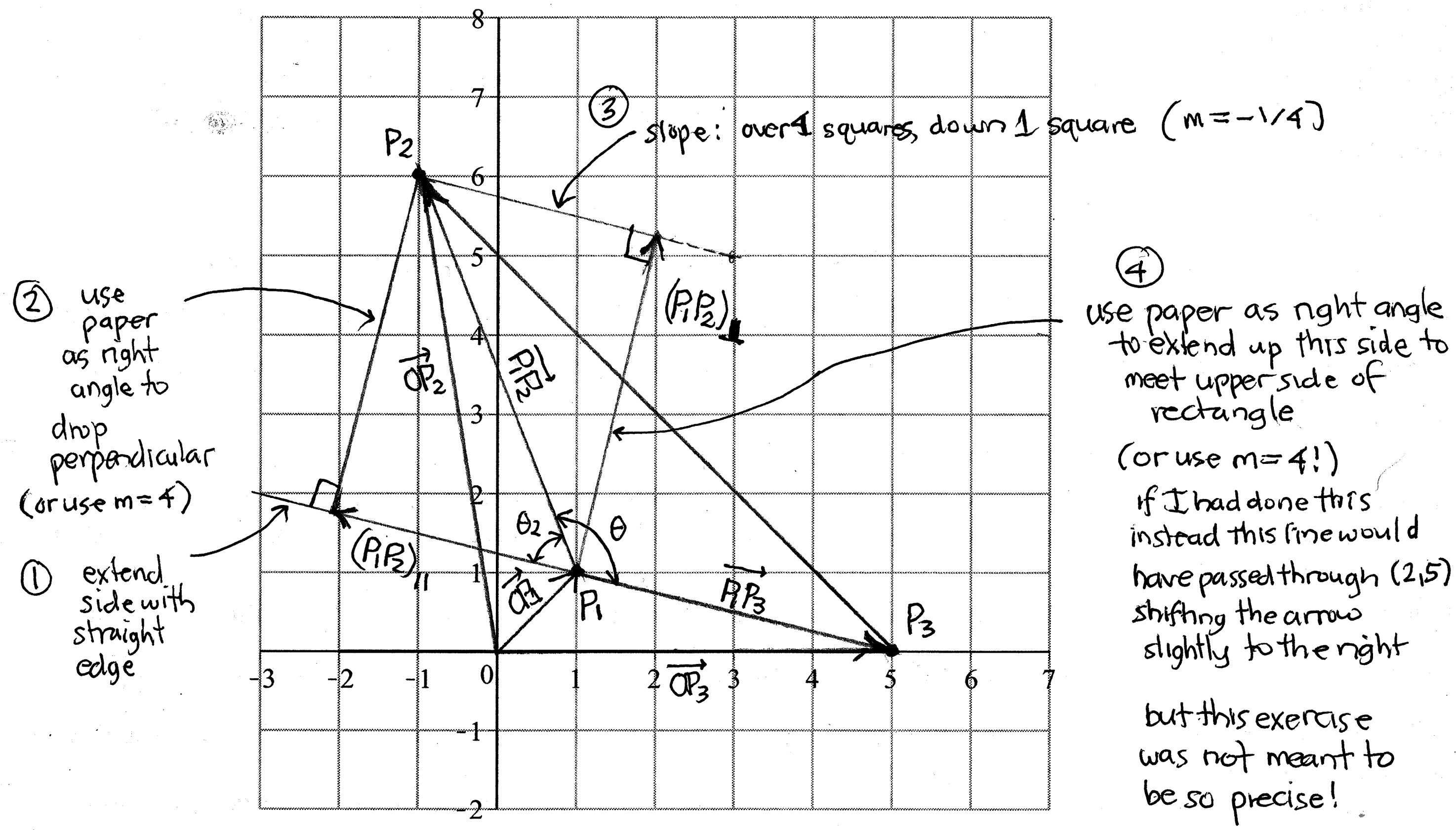
$(\overrightarrow{P_1P_2})_{\perp} = \overrightarrow{P_1P_2} - (\overrightarrow{P_1P_2})_{\parallel}$

$= \langle -2,5\rangle - \left\langle -\frac{52}{17},\frac{13}{17}\right\rangle = \left\langle -2+\frac{52}{17}, 5-\frac{13}{17}\right\rangle$

$= \left\langle \frac{52-34}{17}, \frac{85-13}{17}\right\rangle = \boxed{\left\langle \frac{18}{17},\frac{72}{17}\right\rangle \approx \langle 1.06, 4.24\rangle}$

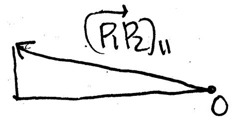
e) compare to:
$\langle 1.0, 4.3\rangle$ also pretty good

$P_1(1,1), P_2(-1,6), P_3(5,0)$



b) θ_2 is a bit less than $60°$ so θ is a bit more than $180°-60° = 120°$, but how much? Maybe $10°$? so $\boxed{\theta \approx 130°}$ guesstimate

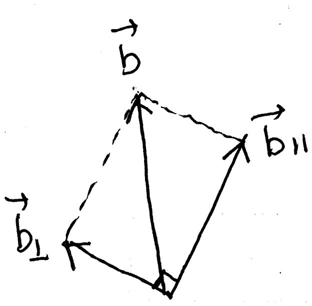
c)



Counting squares and interpolating one estimates right triangle sides:

$$\boxed{\begin{aligned}(\overrightarrow{P_1P_2})_{\parallel} &\approx \langle -3.1, 0.75\rangle \\ (\overrightarrow{P_1P_2})_{\perp} &\approx \langle 1.0, 4.3\rangle\end{aligned}}$$

Note:



a projection diagram rectangle should have a common vertex for its labeled sides $\vec{b}_{\parallel}$ and $\vec{b}_{\perp}$ and its main diagonal $\vec{b}$ so that it represents parallelogram vector addition: $\vec{b} = \vec{b}_{\parallel} + \vec{b}_{\perp}$