MAT2500-03/04 14S Test 2 Answers

① a) $f(x,y,z) = 2xy^2z^2 - y^2 - z^3 + 1 \quad f(1,-1,1) = 2-1-1+1 = 1$

b) $f_x = 2y^2z^2$

$f_y = 4xyz^2 - 2y$

$f_z = 4xy^2z - 3z^2$

level surface: $\boxed{2xy^2z^2 - y^2 - z^3 + 1 = 1}$ or $\boxed{2xy^2z^2 - y^2 - z^3 = 0}$

$\vec{\nabla} f(x,y,z) = \langle 2y^2z^2, 4xyz^2 - 2y, 4xy^2z - 3z^2 \rangle$

$\vec{\nabla} f(1,-1,1) = \langle 2, -4+2, 4-3 \rangle = \langle 2,-2,1 \rangle$

$\hat{u} = \dfrac{\langle 2,-2,1 \rangle}{\sqrt{4+4+1}} = \boxed{\frac{1}{3}\langle 2,-2,1 \rangle}$

$D_{\hat{u}} f(1,-1,1) = |\vec{\nabla} f(1,-1,1)| = \boxed{3}$

c) $\overrightarrow{PO} = -\langle 1,-1,1 \rangle$ from P to O! $(-\overrightarrow{OP})$

$\widehat{PO} = -\dfrac{\langle 1,-1,1 \rangle}{\sqrt{3}}$

$D_{\widehat{PO}} f(1,-1,1) = \widehat{PO} \cdot \vec{\nabla} f(1,-1,1)$

$= -\dfrac{\langle 1,-1,1 \rangle}{\sqrt{3}} \cdot \langle 2,-2,1 \rangle$

$= -\dfrac{(2+2+1)}{\sqrt{3}} = \boxed{\dfrac{-5}{\sqrt{3}}} < 0$ $\boxed{\text{decreasing}}$

no d)!

f) $\vec{n} = \langle 2,-2,1 \rangle$ simplest normal

$\vec{r}_0 = \langle 1,-1,1 \rangle$

$\vec{r} = \vec{r}_0 + t\vec{n} = \langle 1,-1,1 \rangle + t\langle 2,-2,1 \rangle$

$\boxed{\langle x,y,z \rangle = \langle 1+2t, -1-2t, 1+t \rangle}$

or

$\boxed{x = 1+2t,\ y = -1-2t,\ z = 1+t}$

e) $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$

$\langle 2,-2,1 \rangle \cdot \langle x-1, y+1, z-1 \rangle = 0$

$2(x-1) - 2(y+1) + (z-1) = 0$

$2x - 2y + z \ -2-2-1 = 0$

$\boxed{2x - 2y + z = 5}$

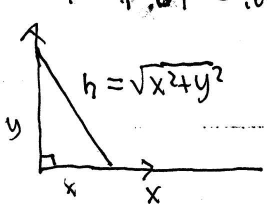
g) $L(x,y,z) = f(1,-1,1) + \vec{\nabla} f(1,-1,1) \cdot (\vec{r} - \vec{r}_0)$

$= 1 + 2(x-1) - 2(y+1) + (z-1)$

$f(1.02, -0.98, 1.03) \approx L(1.02, -0.98, 1.03)$

$= 1 + 2(1.02-1) - 2(-0.98+1) + (1.03-1)$

$= 1 + .04 - .04 + .03 = \boxed{1.03}$

② 

$x = 8,\ y = 15,\ h = \sqrt{64+225} = 17$

$dx = 0.2 \ dy = 0.2$

$dh = \frac{1}{2}(x^2+y^2)^{-1/2}(2x\,dx + 2y\,dy)$

② $dh = \dfrac{x\,dx + y\,dy}{(x^2+y^2)^{1/2}}$

$dh\big|_{x=8,\ y=15} = \dfrac{8\,dx + 15\,dy}{17}$

$dh\big|_{x=8,\ y=15,\ dx=.2,\ dy=.2} = \dfrac{8(.2)+15(.2)}{17} = \dfrac{23}{17}(0.2) = 0.2661$

$\approx \boxed{0.27\text{ cm}}$

$\dfrac{dh}{h} \approx \dfrac{0.2661}{17} \approx \boxed{1.6\%}$

[$dx = \pm 0.2$, $dy = \pm 0.2$ but no cancellation among terms in diff. approximation so $dh \approx \pm 0.27$]

③ $f(x,y) = (x^2 + xy)e^{-y}$

$f_x = (2x+y)e^{-y}$

$f_y = xe^{-y} + (x^2+xy)(-e^{-y}) = (x - x^2 - xy)e^{-y} = x(1-x-y)e^{-y}$

$f_{xx} = 2e^{-y}$

$f_{xy} = 1e^{-y} + (2x+y)(-e^{-y}) = e^{-y}(1-2x-y)$

$f_{yy} = -xe^{-y} + (x-x^2-xy)(-e^{-y}) = e^{-y}(-2x + x^2 + xy)$

$= xe^{-y}(x+y-2)$

a) $f_x(0,0) = 0 = f_y(0,0)$ ✓

$f_x(-1,2) = (-2+2)e^{-2} = 0$, $f_y(-1,2) = -(1+1-2)e^{-2} = 0$ ✓

so both are critical points.

b)

	(0,0)	(-1,2)
f_{xx}	$2>0$ ✓	$2e^{-2}>0$ ✓
f_{yy}	0 ?	$(2+1-2)e^{-2} = e^{-2} > 0$ ✓
f_{xy}	1	$(1+2-2) = e^{-2}$
$f_{xx}f_{yy} - f_{xy}^2$	$0-1<0$ $\boxed{\text{saddle}}$	$(2-1)e^{-4} > 0$

local min? explanation

confirms $\boxed{\text{local min}}$

④ $x + y + z = 12$, $x>0, y>0, z>0$ (constraint)

$S = x^2 + y^2 + z^2$ minimize, eliminate z: $z = 12 - x - y > 0$

$S(x,y) = x^2 + y^2 + (12-x-y)^2$

$S_x = 2x + 2(12-x-y)(-1) = 2(-12+2x+y) = 0$

$S_y = 2y + 2(12-x-y)(-1) = 2(-12+x+2y) = 0$

$2x + y = 12 \to y = 12 - 2x$

$x + 2y = 12 \to \quad x + 2(12-2x) = 12$

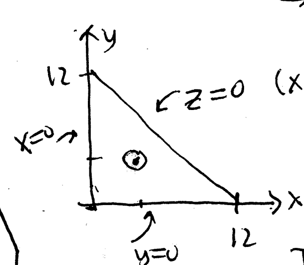
$x + 24 - 4x = 12$

$-3x = -12 \quad x = 4 \to y = 12 - 8 = 4$

$\to z = 12 - 4 - 4 = 4$

$S(4,4) = 3(4^2)$

min value $= \boxed{48}$



interior of triangle $\boxed{x>0, y>0, x+y<12}$

$\boxed{\text{Three positive numbers are all 4.}}$

The point (4,4,4) is the point on the plane $x+y+z=12$ closest to the origin, at a distance $\sqrt{48}$.