MAT2705-01/02 14F Quiz 8 Print Name (Last, First) ______________________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (**but give decimal approximations for interpretation**). Indicate where technology is used and what type (Maple, GC).

+ forgot to edit quiz sign, answer key for minus sign originally

1. $my''+cy'+ky=0$, $y(0)=-3$, $y'(0)=\overset{+}{-}3$; $m=18$, $c=12$, $k=74$. [prime is d/dt]

a) State Maple's solution of the initial value problem.

b) Put the DE into standard linear form first. Then identify the values of the damping constant and characteristic time $k_0=1/\tau_0$, the natural frequency ω_0, and the quality factor $Q=\omega_0\tau_0$, exactly and numerically. Is this underdamped, critically damped or overdamped?

c) Find the general solution by hand, showing all steps.

d) Find the solution satisfying the initial conditions, showing all steps.

e) Give exact and numerical values of the amplitude and phase shift and re-express the sinusoidal factor of this solutio in phase-shifted cosine form. [Make sure you use a diagram to justify your values.] State what numerical fraction of a cycle (2 π) the phase shift is (i.e., evaluate δ/2π) as well as its numerical value in degrees, and whether the cosine curve is shifted left (earlier in time) or right (later in time) on the time line (by a phase less than or equal to half a cycle of course). Explain.

f) State the two envelope functions of this decaying oscillating solution.

g) Make a rough sketch of the plot of your solution and its two envelope functions in a viewing window of width 5 times the characteristic time of the solution exponential factor.

h) What are the numerical values of the periods associated with the natural frequency and the actual frequency of the sinusoidal factor of the solution and how do they compare?

▶ solution (see note above)

① a) $y(t) = 1\,e^{-\frac{1}{3}t}\sin(2t) - 3e^{-\frac{1}{3}t}\cos(2t)$

b) $18y''+12y'+74y=0$ divide by 2!

↳ $9y''+6y'+37y=0$ divide by 9.

$\boxed{y''+\frac{2}{3}y'+\frac{37}{9}y=0}$

$k_0=\frac{2}{3}\to\tau_0=\frac{3}{2}=1.5$ $\quad\omega_0=\frac{\sqrt{37}}{3}\approx 2.03$

$Q=\frac{3}{2}\frac{\sqrt{37}}{3}=\frac{\sqrt{37}}{2}\approx 3.04 > \frac{1}{2}$ so underdamped

c) $y=e^{rt}$: → $9r^2+6r+37=0$

MAPLE: ↳ $r=-\frac{1}{3}\pm 2i$

$e^{rt}=e^{-t/3}e^{\pm 2it}=e^{-\frac{t}{3}}(\cos 2t \pm i\sin 2t)$

↳ real basis: $e^{-t/3}\cos 2t,\ e^{-t/3}\sin 2t$

$\boxed{y=e^{-t/3}(c_1\cos 2t+c_2\sin 2t)}$ gen. soln.

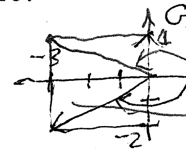
d) $y'=-\frac{1}{3}e^{-t/3}(c_1\cos 2t+c_2\sin 2t)$
$\quad +e^{-t/3}(-2c_1\sin 2t+2c_2\cos 2t)$

$y(0)=c_1=-3$

$y'(0)=-\frac{1}{3}c_1+2c_2=+3\to c_2=\frac{1}{2}(+3+\frac{1}{3}(-3))=1$

$\boxed{y=e^{-t/3}(-3\cos 2t+1\sin 2t)}$ IVP soln.

e)



$\delta=\pi-\arctan\left(\frac{1}{3}\right)$ ~~≈ 3.46 ≈ 198°~~ ~~≈ 0.55 cycles~~
$\approx 2.82 \approx 162° \approx 0.45$ cycles

~~$\delta=-\pi+\arctan 2/3 \approx -2.55 \approx -146°$~~
~~≈ -0.41 cycles~~

$A=\sqrt{1+9}=\sqrt{10}\approx$ ~~3.61~~ 3.16

$\boxed{y=\sqrt{10}\cos(2t+\pi-\arctan 2/3)\,e^{-t/3}}$

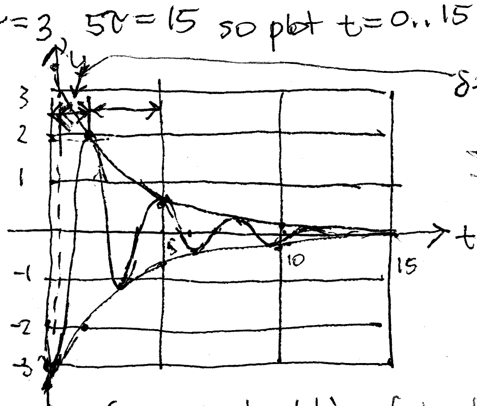
negative obtuse angle so shifted left (earlier in time) compared to cos 2t.

f) $A(t)=\sqrt{10}\,e^{-t/3}$ → $\boxed{y=\pm\sqrt{10}\,e^{-t/3}}$ envelope curves

h) $T_0=\frac{2\pi}{\omega_0}=\frac{6\pi}{\sqrt{37}}\approx 3.099\approx 3.10$

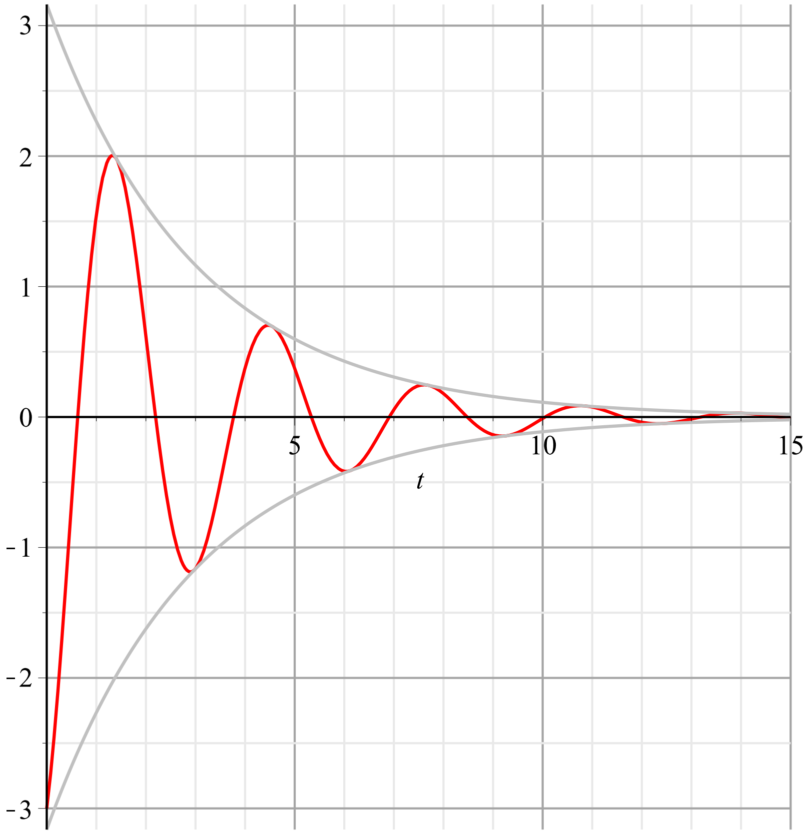
$T=\frac{2\pi}{2}=\pi\approx 3.14$ — ↑ slightly longer due to damping which slows down system.

g) $\tau=3$, $5\tau=15$ so plot $t=0..15$:

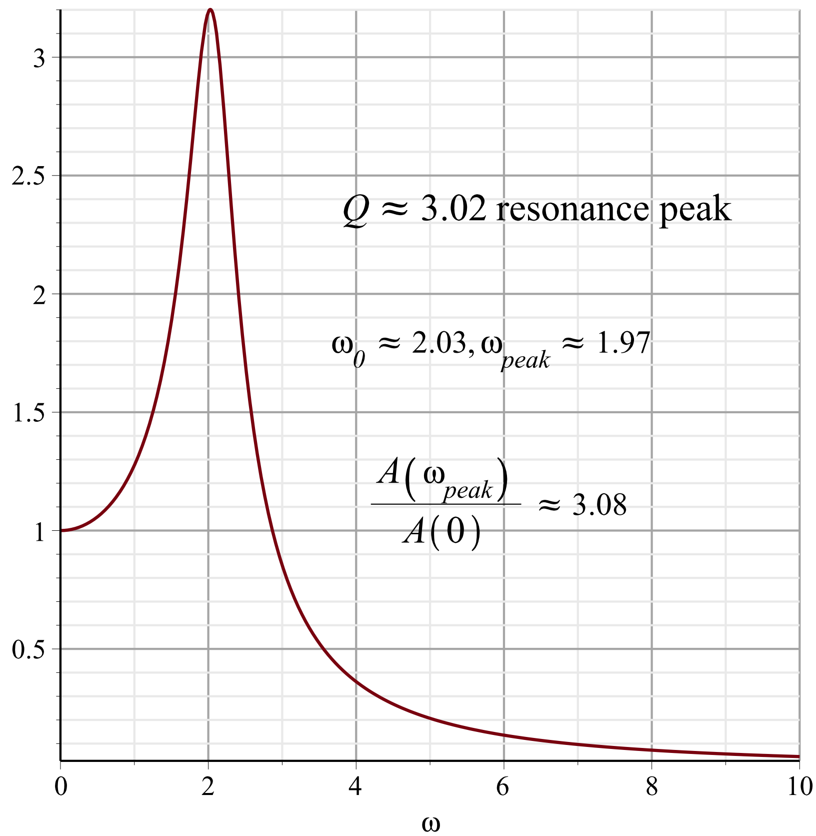


0.45 cycles right.
~~δ ≈ .4 cycles (left)~~
~~less than 1/2 cycle~~
~~so about .6 cycles to right to first peak~~
~~.6T ~ 1.7~~
~~compared to period T ~ 3.14~~

(see Maple plot) fortunately hand sketch not precise enough to see difference with correction



3
2
1
0
-1
-2
-3
5
10
15
t


$Q \approx 3.02$ resonance peak
$\omega_0 \approx 2.03, \omega_{peak} \approx 1.97$
$\frac{A(\omega_{peak})}{A(0)} \approx 3.08$
ω
0
2
4
6
8
10
0.5
1
1.5
2
2.5
3