MAT2705-01/02 14F Quiz 7 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $6y''+5y'+y=0,\ y(0)=1,\ y'(0)=2$.

a) Find the general solution $y(x)$ of the DE.

b) Using 2x2 matrix methods find the solution which satisfies the initial conditions, showing all work.

c) What are the two characteristic lengths $\tau_1<\tau_2$ for the two decaying exponentials in this problem? Let $\tau=\max(\tau_1,\tau_2)$.

d) Use technology to plot your result for $x=0..5\,\tau$ and an appropriate vertical window including the horizontal axis and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch. Locate on your sketch and label the maximum point. (Clicking on the gridlines icon for a Maple plot helps you make your hand sketch more accurate.)

e) Use calculus to determine ***exactly*** by hand (rules of exponents and logs! simplify your result for y to a simple term) the x and y values of the obvious maximum point on the graph and then their approximate values to 4 decimal places. Do the numbers you found agree with what your eyes see in the technology plot? [Yes or no, with an explanation would be a good response.]

▶ solution

1 a) $6y''+5y'+y=0 \to 6r^2e^{rx}+5re^{rx}+e^{rx}=0$

$y=e^{rx}$, $y'=re^{rx}$, $y''=r^2e^{rx}$

$(6r^2+5r+1)e^{rx}=0$

$6r^2+5r+1=0$

$r=\dfrac{-5\pm\sqrt{25-4(6)(1)}}{2\cdot 6}=\dfrac{-5\pm1}{12}=-\dfrac{6}{12},\dfrac{-4}{12}=-\dfrac12,-\dfrac13$

$e^{rx}=e^{-x/2},\ e^{-x/3}$

$\boxed{y=c_1e^{-x/2}+c_2e^{-x/3}}$ (gen soln)

b) $y'=-\dfrac{c_1}{2}e^{-x/2}-\dfrac{c_2}{3}e^{-x/3}$

$y(0)=c_1+c_2=1$

$y'(0)=-\dfrac{c_1}{2}-\dfrac{c_2}{3}=2$

$$\begin{bmatrix}1 & 1\\ -\frac12 & -\frac13\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix},\quad \begin{bmatrix}c_1\\ c_2\end{bmatrix}=\frac{1}{-\frac13+\frac12}\begin{bmatrix}-\frac13 & -1\\ \frac12 & 1\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}$$

$$=6\begin{bmatrix}-\frac13-2\\ \frac12+2\end{bmatrix}=\begin{bmatrix}-\frac73\cdot 6\\ \frac52\cdot 6\end{bmatrix}=\begin{bmatrix}-16\\ 15\end{bmatrix}$$

oops, mental error $+\frac13+2=\frac{6+1}{3}=\frac73$ I put 8/3 originally!

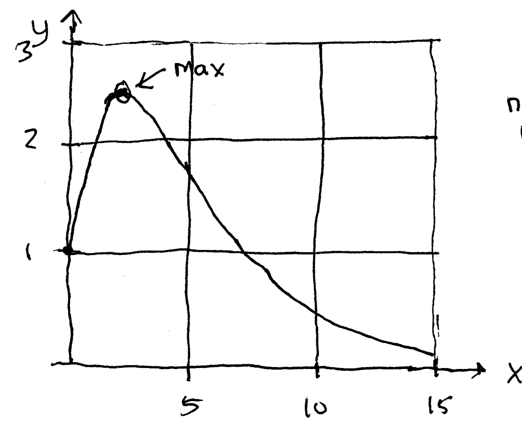
$\boxed{y=-14e^{-x/2}+15e^{-x/3}}$ (IVP soln)

c) $k_1=\frac12$, $k_2=\frac13$

$\boxed{\tau_1=1/k_1=2,\ \tau_2=1/\tfrac13=3}$

$\boxed{\tau=3}$

d) $5\tau=15$ so $x=0..15$



not so pretty!

e) $y'=-14(-\frac12)e^{-x/2}+15(-\frac13)e^{-x/3}=0$

$[7e^{-x/2}=5e^{-x/3}]\,e^{x/2}$

$7=5e^{x/6}$

$e^{x/6}=7/5$, $\frac{x}{6}=\ln\frac75$, $\boxed{x=6\ln\frac75}$ $\boxed{\approx 2.0188}$

$y=-14e^{-\frac62\ln 7/5}+15e^{-\frac63\ln 7/5}$

$=-14e^{-3\ln 7/5}+15e^{-2\ln 7/5}$

$=-14e^{\ln(7/5)^{-3}}+15e^{\ln(7/5)^{-2}}$

$=-14\left(\frac75\right)^{-3}+15\left(\frac75\right)^{-2}$

$=-14\left(\frac57\right)^{3}+15\left(\frac57\right)^{2}$

$=-\dfrac{2(125)}{49}+\dfrac{15\cdot 25}{49}=\cancel{\dfrac{25(5-2)}{49}=\dfrac{75}{49}}$

$=\dfrac{(3-2)125}{49}=\boxed{\dfrac{125}{49}}=y$ $\boxed{\approx 2.5510}$

on grid x is a hair above 2, y is a hair above 2.5 so yes this agrees.