MAT2705-01/02 14F Quiz 3 Print Name (Last, First) _____

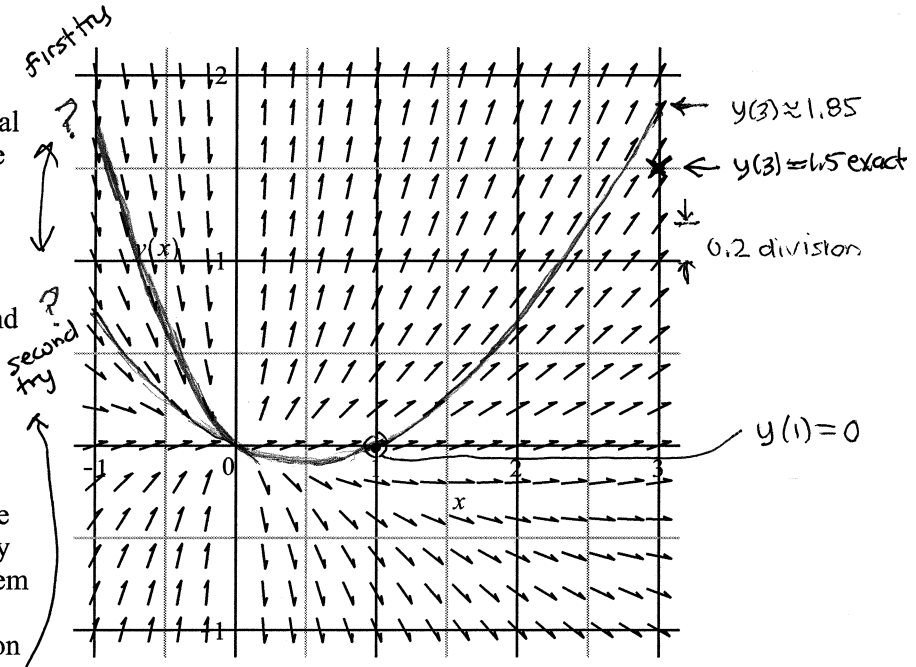
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $x\frac{dy}{dx} - 2y - \frac{x}{4} = 0,\ y(1) = 0,\ x \geq 0.$

a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right. Put a circled dot at the point corresponding to the initial condition, annotated by an arrow to its location from the label $y(1) = 0$. Similarly indicate the point on this curve where $x = 3$ and estimate $y(3)$. Don't change your estimated value after you later evaluate the solution exactly.

b) Use the linear solution recipe to find the general solution of this differential equation. Simplify it and box it. What is the name of the shape of the curves which make up this family of solutions? Does your hand drawn curve seem to have this shape? Explain.

c) Find the solution of this differential equation which satisfies the given initial condition.



a) $x=0$: $x\frac{dy}{dx} - 2y - \frac{x}{4} = 0$ → $y=0$ must pass thru origin

not possible to guess how curve emerges to left after passing through the origin! UNSTABLE POINT!

d) Evaluate your solution at $x = 3$ numerically to 2 decimal places and mark the corresponding point on your graph with a visible ×. Is this consistent with your part a) result? Explain.

e) Does your initial value problem solution agree with Maple ? If equivalent, show the equivalence. If not, can you find your mistake?

▶ solution

① b) $\left[\frac{dy}{dx} - \frac{2}{x}y = \frac{1}{4}\right]$ (divide by x; put constant term on RHS)

$\int -\frac{2}{x}dx = -2\ln|x|$

$e^{\int -\frac{2}{x}dx} = e^{\ln|x|^{-2}} = x^{-2}$ (integrating factor)

$x^{-2}\left(\frac{dy}{dx} - \frac{2}{x}y\right) = \frac{1}{4}x^{-2}$ (integrate both sides)

$\frac{d}{dx}(x^{-2}y) = \frac{1}{4}x^{-2}$

$x^{-2}y = \int \frac{1}{4}x^{-2}dx = \frac{1}{4}\frac{x^{-1}}{-1} + C = -\frac{1}{4}x^{-1} + C$

$y = x^2\left(-\frac{1}{4}x^{-1} + C\right) = -\frac{1}{4}x + Cx^2$ gen soln

graph is a parabola (all pass through origin)

my first attempt did not, then my second did (prejudiced by this knowledge)

c) $0 = y(1) = -\frac{1}{4} + C \rightarrow C = \frac{1}{4}$

$y = -\frac{1}{4}x + \frac{1}{4}x^2 = \frac{1}{4}(x^2 - x)$ IVP soln.

d) $y(3) = \frac{1}{4}(3^2 - 3) = \frac{6}{4} = \frac{3}{2} = 1.5$

perhaps my initial slope was a bit high, which pulled my curve to the left & hence hit the endpoint high. Oh well – it is a rough graph – what can you expect?

e) Maple gives: $y(x) = -\frac{1}{4}x + \frac{1}{4}x^2$, same as hand result.