MAT2705-01/02 14F Quiz 2 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Always justify your claims.

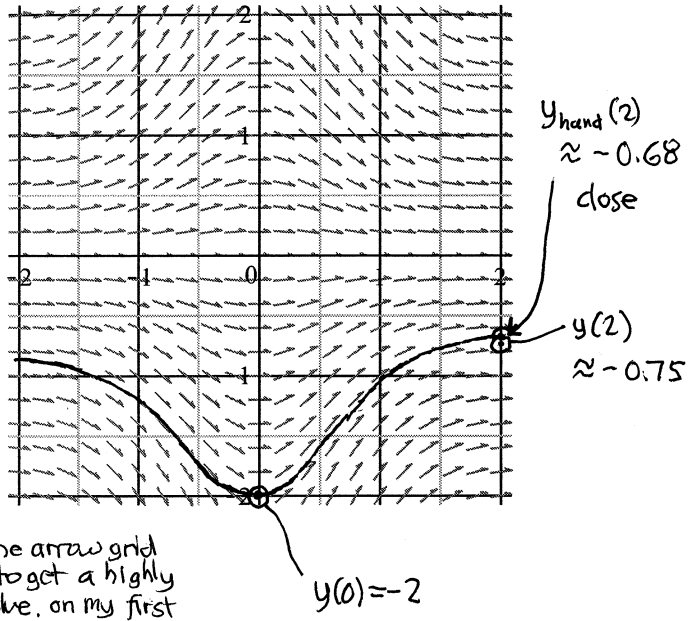
1. $\frac{dy}{dx} + 2yxe^{-x^2} = 0, y(0) = -2.$

a) Indicate the initial data point on the graph by a circled dot annotated by an arrow pointing to the point from the initial condition written to the side of the graph and roughly draw in the corresponding solution curve.

b) Find the general solution of this separable differential equation, making sure you redefine your final constant to include the obvious missing solution from your family.

c) Find the solution which satisfies the initial condition (no decimal point numbers!). Simplify your result by combining the exponentials.

d) Evaluate $y(2)$ for this solution and mark the corresponding point on the graph by a circled dot annotated as above (arrow from $y(2) \approx ...$). Is this consistent with your approximate hand drawn solution? Explain.



the resolution of the arrow grid makes it difficult to get a highly accurate endpoint value. on my first attempt my curve was even farther up.

e) Check by hand that your solution to c) solves the differential equation. [Remember, backsub everywhere in the DE eliminating y, then simplify both sides independently.]

f) Enter the differential equation and the initial condition separated by a comma in Maple. Right click and solve. Write down exactly the form of the solution that it gives you. Does it agree with your hand solution? Explain why if so. If not, you better find your error.

▶ solution

(1b) $\frac{dy}{dx} = -2yxe^{-x^2}$ (separate)

$\int \left[\frac{1}{y}dy = -2xe^{-x^2}\right]dx$ (integrate) $y \neq 0$!

$\int \frac{dy}{y} = \int -2xe^{-x^2}dx \overset{du}{=} \int e^u du = e^u + C_1 = e^{-x^2} + C_1$

$\ln|y|$

e^{u}

$u = -x^2$

$du = -2x\,dx$

$e^{[\ln|y| = e^{-x^2} + C_1]}$ (exponentiate) (use parentheses if necessary to keep C_1 in exponent!)

$|y| = e^{(e^{-x^2}+C_1)} = e^{C_1}e^{e^{-x^2}}$

$y = \underbrace{\pm e^{C_1}}_{\equiv C} e^{e^{-x^2}} \rightarrow \boxed{y = Ce^{e^{-x^2}}}$ gen soln (allows $y=0$)

c) $y(0) = -2 \leftrightarrow x = 0, y = -2$: $-2 = Ce^{e^{-0}} = Ce^{1} = Ce$

$C = -\frac{2}{e}$: $y = -\frac{2}{e}e^{e^{-x^2}} \rightarrow \boxed{y = -2e^{(e^{-x^2}-1)}}$ combine exps!

d) $y(2) = -2e^{(e^{-x^2}-1)} \approx -0.74936$

my hand curve veered up a bit but close enough for this resolution grid.

e) $y = -2e^{e^{-x^2}-1}$

$\frac{dy}{dx} = -2e^{e^{-x^2}-1}\underbrace{\frac{d}{dx}(e^{-x^2}-1)}_{-2xe^{-x^2}}$

$= 4xe^{(e^{-x^2}-1)}e^{-x^2}$

subs into DE

$4xe^{(e^{-x^2}-1)}e^{-x^2} = -2\left(-2e^{e^{-x^2}-1}\right)xe^{-x^2}$

$= 4e^{(e^{-x^2}-1)}e^{-x^2}$ ✓

f) Maple gives: $y = -2e^{e^{-x^2}}$

which is exactly what I got before combining exps!