MAT2705-01/02 14f Quiz 1 Print Name (Last, First) ________________, ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y' = e^{-y}$: *gen sol* : $y(x) = \ln(x+C)$

a) Verify that this y satisfies the given differential equation.

b) Find the solution which satisfies the initial condition $y(0) = 0$.

Organize your work as though you were playing professor.

you must be able to read intelligently!

2. Choose appropriately named variables and write a differential equation that models the situation:
"When sugar is dissolved in water the time rate of change of the amount that remains undissolved in water is proportional to that amount"
Knowing that this amount should decrease, what is the sign of the constant of proportionality you introduced?

▶ solution

① a) $y = \ln(x+C)$

$$\frac{dy}{dx} = \frac{1}{x+C}\underbrace{\frac{d}{dx}(x+C)}_{1+0} = \frac{1}{x+C}$$

$$\boxed{\frac{dy}{dx} = e^{-y}} \rightarrow \frac{1}{x+C} = e^{-\ln(x+C)} = \left(e^{\ln(x+C)}\right)^{-1} = (x+C)^{-1} = \frac{1}{x+C} \checkmark$$

b) $y(0)=0 \Leftrightarrow x=0, y=0$

$y = \ln(x+C)$

$[0 = \ln(0+C) = \ln C]$ ← solve by exponentiating both sides

$e^0 = e^{\ln C} = C$

$e^0 = 1$

$\therefore C = 1$

$$\boxed{y = \ln(x+1)}$$

always backsub values of constants!!

② A = Amount of sugar undissolved ≥ 0

$$\frac{dA}{dt} \propto A$$

$$\underbrace{\frac{dA}{dt} = -kA}_{\leq 0}, \quad k > 0$$

≤ 0 corresponds to decreasing amount

$$\left(\text{or } \frac{dA}{dt} = kA, \quad k<0 \right)$$

[always support your claims with some kind of reasoning ~ use words if necessary]

To check the solution of any equation(s), simply replace the unknown(s) by the expression(s) for its (their) solution everywhere in the equation(s) and simplify independently the LHS and RHS without performing any other operations. If the two sides simplify to the same expression, the solution is confirmed. Otherwise, not.