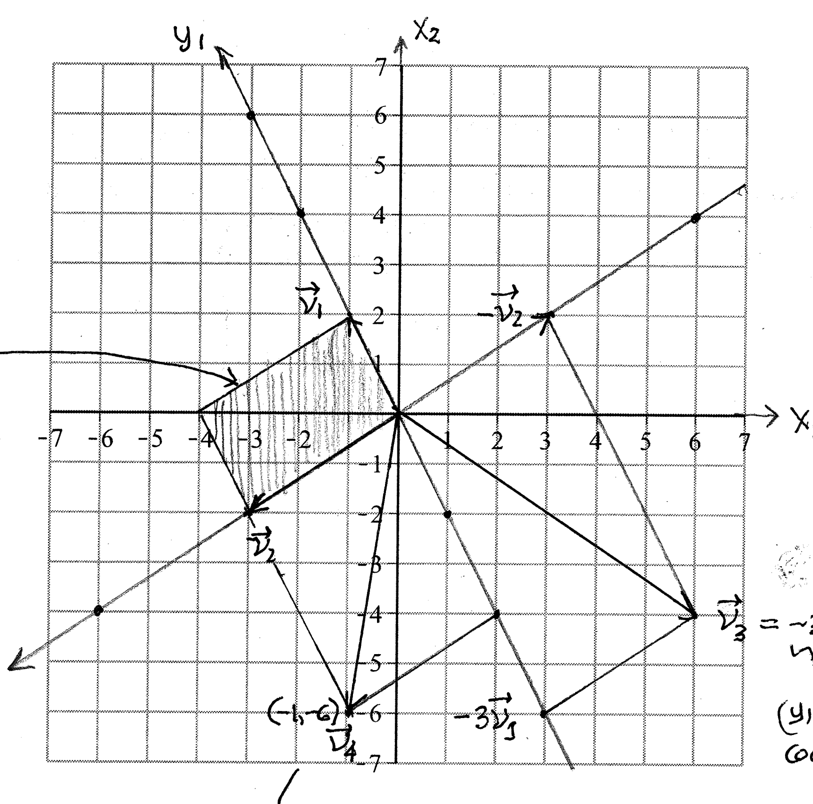
MAT2705-01/02 14F Test 2 Print Name (Last, First) ________________________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses.**

1. a) On the grid below, **draw in** arrows representing the vectors $\vec{v}_1 = \langle -1, 2\rangle$ and $\vec{v}_2 = \langle -3,-2\rangle$ and $\vec{v}_3 = \langle 6,-4\rangle$ and **label** them by their symbols. Then **draw in** the parallelogram that graphically expresses $\vec{v}_3$ as a linear combination of $\{\vec{v}_1, \vec{v}_2\}$. **Label** its two sides that intersect at the origin by the corresponding vectors they represent. **Extend** the basis vectors $\{\vec{v}_1, \vec{v}_2\}$ to the corresponding coordinate axes for (y_1, y_2) and **mark** the positive direction with an arrow head and the axis label. Finally shade in the region of the plane corresponding to the unit parallelogram associated with the new coordinates: $0 \le y_1 \le 1, 0 \le y_2 \le 1$.
Draw in the projection parallelogram associated with the new coordinate system whose main diagonal is $\vec{v}_3$, then read off the coordinates (y_1, y_2) of $\vec{v}_3$ with respect to these two vectors (write them down) and **express** $\vec{v}_3$ as a linear combination of these vectors; **put this equation** at the tip of this vector. **Explain** how you got these numbers.
b) Now write down the matrix equation that enables you to express $\vec{v}_3$ as a linear combination of the other two vectors, solve that system using matrix methods, and then express $\vec{v}_3$ explicitly as a linear combination of those vectors.
c) Check your linear combination by expanding it out to get the original vector. Did you (get the original vector)? Does your matrix result agree with part a)?
d) Now using the new coordinate axes, **draw in** the vector $\vec{v}_4$ whose new coordinates are $(y_1, y_2) = (-2, 1)$ and **label** the tip of $\vec{v}_4$ by its symbol. Then draw in the projection parallelogram associated with the new coordinates for which v_4 is the main diagonal. Read off its old coordinates (x_1, x_2) from the grid. Do they agree with the linear combination $y_1\vec{v}_1 + y_2\vec{v}_2$?

① a)

$0 \le y_1 \le 1,$
$0 \le y_2 \le 1$



$\vec{v}_3 = -3\vec{v}_1 - \vec{v}_2 = \langle 6,-4\rangle$ (given)

$(y_1,y_2) = (-3,-1)$ coordinate readoff

✓

d) $\vec{v}_4 = -2\vec{v}_1 + \vec{v}_2 = \underbrace{\langle -1,-6\rangle}_{=\langle x_1, x_2\rangle}$ graphical readoff

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(2) a) $A\vec{x}=\vec{0}$: $\begin{bmatrix} 2 & 3 & 8 & 0 \\ -1 & 4 & 7 & 1 \\ 4 & 2 & 8 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\langle A|\vec{0}\rangle = \begin{bmatrix} 2 & 3 & 8 & 0 & 0 \\ -1 & 4 & 7 & 1 & 0 \\ 4 & 2 & 8 & -2 & 0 \end{bmatrix} \xrightarrow[\text{Maple}]{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 & \\ L & L & F & L & \end{matrix} \end{array}$

$x_3 = t$

$x_1 + x_3 = 0 \quad x_1 = -t$

$x_2 + 2x_3 = 0 \quad x_2 = -2t$

$x_4 = 0$

$$\boxed{\vec{x} = \begin{bmatrix} -t \\ -2t \\ t \\ 0 \end{bmatrix}} = t\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} \equiv t\vec{u}_1, \quad \vec{u}_1 = \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

b) basis of soln subspace $\boxed{\{\langle -1,-2,1,0\rangle\}}$

c) $\boxed{-1\vec{v}_1 - 2\vec{v}_2 + 1\vec{v}_3 + 0\vec{v}_4 = \vec{0}}$ ←

d) One relationship among 4 vectors so 3 are linearly independent — one can choose the leading columns as a linearly independent subset so the span of these vectors is all of $\mathbb{R}^3$ and $\{\vec{v}_1, \vec{v}_2, \vec{v}_4\}$ is a basis of $\mathbb{R}^3$.

[In fact $\vec{v}_4$ plus any 2 vectors from $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ which span a plane will be a basis of $\mathbb{R}^3$]

e) $A\vec{x} = \vec{v}_5$: $\begin{bmatrix} 2 & 3 & 8 & 0 \\ -1 & 4 & 7 & 1 \\ 4 & 2 & 8 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix}$

$\langle A|\vec{v}_5\rangle = \begin{bmatrix} 2 & 3 & 8 & 0 & 1 \\ -1 & 4 & 7 & 1 & -5 \\ 4 & 2 & 8 & -2 & 4 \end{bmatrix} \xrightarrow[\text{Maple}]{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 & \\ L & L & F & L & \end{matrix} \end{array}$

$x_3 = t$

$x_1 + x_3 = 2 \rightarrow x_1 = 2 - t$

$x_2 + 2x_3 = -1 \quad x_2 = -1 - 2t$

$x_4 = 1 \quad x_4 = 1$

$$\boxed{\vec{x} = \begin{bmatrix} 2-t \\ 1-2t \\ t \\ 1 \end{bmatrix}} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\vec{v}_5 = (2-t)\vec{v}_1 + (-1-2t)\vec{v}_2 + t\vec{v}_3 + \vec{v}_4}$$

$$\boxed{\overset{t=0}{=} 2\vec{v}_1 - \vec{v}_2 + \vec{v}_4}$$

(2) f) repeat with $\vec{v}_5 = \langle 1,2,4\rangle$

$\langle A|\vec{v}_5\rangle \xrightarrow[\text{Maple}]{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 & \\ L & L & F & L & \end{matrix} \end{array}$

$x_3 = t$

$x_1 = -1 - t$

$x_2 = 1 - 2t$

$x_4 = -3$

$\vec{x} = \begin{bmatrix} -1-t \\ 1-2t \\ t \\ -3 \end{bmatrix} \rightarrow$

$$\boxed{\vec{v}_5 = (-1-t)\vec{v}_1 + (1-2t)\vec{v}_2 + t\vec{v}_3 - 3\vec{v}_4}$$

$$\boxed{\overset{t=0}{=} -\vec{v}_1 + \vec{v}_2 - 3\vec{v}_4}$$

(1) b) $y_1\begin{bmatrix} -1 \\ 2 \end{bmatrix} + y_2\begin{bmatrix} -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} -1 & -3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}}$

$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 2 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ -4 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -2 & 3 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ -4 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -12-12 \\ -12+4 \end{bmatrix}$

$= \begin{bmatrix} 24/8 \\ -8/8 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$ so $\boxed{\vec{v}_3 = -3\vec{v}_1 - \vec{v}_2}$

c) Check: $-3\begin{bmatrix} -1 \\ 2 \end{bmatrix} - \begin{bmatrix} -3 \\ -2 \end{bmatrix} = \begin{bmatrix} +3+3 \\ -6+2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$ ✓

success, yes, it gives the original vector and it agrees with the graphical result.

d) $y_1\vec{v}_1 + y_2\vec{v}_2 = -2\begin{bmatrix} -1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 2-3 \\ -4-2 \end{bmatrix} = \begin{bmatrix} -1 \\ -6 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

yes this agrees with the graphical result.

a) see graph

$\vec{v}_3 = -3\vec{v}_1 - \vec{v}_2$

1 tickmark along negative y_2 axis (= 1 multiple of $-\vec{v}_2$)

3 tickmarks along negative y_1 axis (= 3 multiples of $-\vec{v}_1$)

coefficients are new coords:

$$\boxed{(y_1, y_2) = (-3,-1)}$$