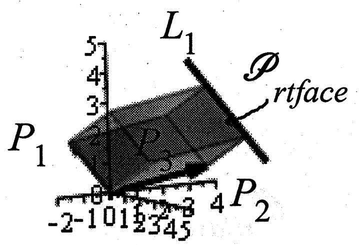
MAT2500-01 13S Quiz 2   Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

Given three points $P_1(1,-2,2)$, $P_2(3,2,1)$, $P_3(1,1,1)$ and the parallelopiped formed from their three position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$. [Note $\vec{r}_1, \vec{r}_2$ form the front face of this object, with $\vec{r}_3$ extending backwards from this face.]



a) Write the parametrized equations of the line L_1 through the back right edge of the parallelopiped as shown. [What is the simplest position vector of a point on the line? What is the orientation of the line?] Where does this line intersect the $z=0$ plane?

b) Find a normal vector $\vec{n}$ for the plane $\mathscr{P}_{rtface}$ which contains the right side face of the parallelepiped shown in the figure.

c) Write the simplified equation for this plane. Does the point at the tip of the main diagonal of the parallelopiped ((from the origin to the opposite corner) satisfy this equation as it should?

d) Find the scalar projection h of the main diagonal of the parallelopiped along $\vec{n}$. [This is just the distance of that right face plane from the origin, or its height if we instead think of that face as the top of the parallelopiped.]

e) Evaluate the area A of the left side face of the parallelogram, a parallelogram formed by the edges $\vec{r}_1, \vec{r}_3$.

f) Does the volume $V=|h|\,A$ of the parallelopiped equal the triple scalar product $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)|$ as it should?

a) $\vec{r} = \vec{r}_0 + t\vec{a}$ $\vec{r}_1 = \langle 1,-2,2\rangle$

$\vec{r}_0 = \vec{r}_2 + \vec{r}_3 = \langle 3,2,1\rangle + \langle 1,1,1\rangle = \langle 4,3,2\rangle$

$\boxed{\langle x,y,z\rangle = \langle 4,3,2\rangle + t\langle 1,-2,2\rangle = \langle 4+t, 3-2t, 2+2t\rangle}$ or $\boxed{x=4+t,\ y=3-2t,\ z=2+2t}$

$z = 2+2t = 0$

$t=-1 \rightarrow x = 4+(-1) = 3$, $y = 3-2(-1) = 5$

$\langle x,y,z\rangle = \langle 3,5,0\rangle$ $\boxed{\text{pt } (3,5,0)}$

b) $\vec{n} = \vec{r}_3 \times \vec{r}_1 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -2 & 2 \end{vmatrix} = \langle 2+2, 1-2, -2-1\rangle = \langle 4,-1,-3\rangle$

c) $\vec{r}_0 = \vec{r}_2 = \langle 3,2,1\rangle$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 4,-1,-3\rangle \cdot \langle x-3, y-2, z-1\rangle$

$= 4(x-3) - (y-2) - 3(z-1) = 4x - y - 3z \underbrace{-12+2+3}_{-7}$

$\rightarrow \boxed{4x-y-3z=7}$

or

$\vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 3 & 2 & 1 \end{vmatrix} = \langle -2-4, 6-1, 2+6\rangle = \langle -6,5,8\rangle$

$r_3 \cdot (\vec{r}_1 \times \vec{r}_2) = \langle 1,1,1\rangle \cdot \langle -6,5,8\rangle = -6+5+8 = 7$

d) $\vec{R} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 = \langle 1,-2,2\rangle + \langle 4,3,2\rangle = \langle 5,1,4\rangle$ main diag.

$\hat{n} = \dfrac{\langle 4,-1,-3\rangle}{\sqrt{16+1+9}} = \dfrac{\langle 4,-1,-3\rangle}{\sqrt{26}}$

$\hat{n}\cdot\vec{R} = \dfrac{\langle 4,-1,-3\rangle}{\sqrt{26}} \cdot \langle 5,1,4\rangle = \dfrac{20-1-12}{\sqrt{26}} = \boxed{\dfrac{7}{\sqrt{26}} = h}$

$|\vec{r}_3 \times \vec{r}_1| = \sqrt{16+1+9} = \boxed{\sqrt{26} = A}$

$\cancel{A = \sqrt{2}}$

f) $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)| = |\vec{r}_2 \cdot (\vec{r}_3 \times \vec{r}_1)|$

$\vec{r}_2 \cdot (\vec{r}_3 \times \vec{r}_1) = \langle 3,2,1\rangle \cdot \langle 4,-1,-3\rangle$

$= 12-2-3 = \boxed{7 = V}$

$hA = \dfrac{7}{\sqrt{26}} \cdot \sqrt{26} = 7 = V$ ✓

▶ solution