MAT2500-01 13S TEST 1 ANSWERS

a) $\vec{r} = \langle e^t, e^{-t}, \sqrt{2}t\rangle$

$z=\sqrt{2}t$ increases with $t \to$ moving up

$$\boxed{\begin{aligned}\vec{r}\,' &= \langle e^t, -e^{-t}, \sqrt{2}\rangle\\ \vec{r}\,'' &= \langle e^t, e^{-t}, 0\rangle\end{aligned}}$$

$$|\vec{r}\,'| = \sqrt{e^{2t}+2+e^{-2t}} = \sqrt{(e^t+e^{-t})^2} = \boxed{e^t+e^{-t}}$$

$$\boxed{|\vec{r}\,''| = \sqrt{e^{2t}+e^{-2t}}}$$

$$\hat{T} = \frac{1}{e^t+e^{-t}}\langle e^t, -e^{-t}, \sqrt{2}\rangle$$

$$\vec{r}(0) = \langle e^0, e^{-0}, 0\rangle = \boxed{\langle 1,1,0\rangle}$$

$$\vec{r}\,'(0) = \langle e^0, -e^{-0}, \sqrt{2}\rangle = \boxed{\langle 1,-1,\sqrt{2}\rangle}$$

$$\vec{r}\,''(0) = \langle e^0, e^{-0}, 0\rangle = \boxed{\langle 1,1,0\rangle}$$

$$|\vec{r}\,'(0)| = \sqrt{e^0+e^{-0}+2}\ \sqrt{4} = \boxed{2}$$

$$\hat{T}(0) = \boxed{\frac{1}{2}\langle 1,-1,\sqrt{2}\rangle}$$

$$|\vec{r}\,''(0)| = \boxed{\sqrt{2}}$$

b) $$\vec{r}\,'\times\vec{r}\,'' = \begin{vmatrix} i & j & k\\ e^t & -e^{-t} & \sqrt{2}\\ e^t & e^{-t} & 0\end{vmatrix}$$

$$= \langle 0-\sqrt{2}e^{-t}, \sqrt{2}e^t-0, e^{t-t}+e^{t-t}\rangle$$

$$= \langle -\sqrt{2}e^{-t}, \sqrt{2}e^t, 2\rangle = \sqrt{2}\langle -e^{-t}, e^t, \sqrt{2}\rangle$$

$$|\vec{r}\,'\times\vec{r}\,''| = \sqrt{2e^{-2t}+2e^{2t}+4}$$ ← even, factor out 2!

$$= \sqrt{2}\underbrace{\sqrt{e^{2t}+2+e^{-2t}}}_{\text{same perfect square}} = \sqrt{2}(e^t+e^{-t})$$

$$\hat{B} = \frac{\vec{r}\,'\times\vec{r}\,''}{|\vec{r}\,'\times\vec{r}\,''|} = \boxed{\frac{1}{e^t+e^{-t}}\langle -e^{-t}, e^t, \sqrt{2}\rangle}$$

c) normal $\vec{n} \propto \vec{B}$ : $\vec{n} = \langle -e^{-0}, e^0, \sqrt{2}\rangle = \langle -1,1,\sqrt{2}\rangle$

reference pt $r_0 = \vec{r}(0) = \langle 1,1,0\rangle$

$$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle -1,1,\sqrt{2}\rangle\cdot\langle x-1, y-1, z-0\rangle$$

$$= -(x-1)+(y-1)+\sqrt{2}z = -x+y+\sqrt{2}z+1-1$$

$$\boxed{-x+y+\sqrt{2}z = 0}$$

d) $\vec{r}_0 = \vec{r}(0)$, $a = \vec{r}\,'(0)$

$$\vec{r} = \vec{r}_0 + t\vec{a} = \langle 1,1,0\rangle + t\langle 1,-1,\sqrt{2}\rangle$$

$$= \boxed{\langle 1+t, 1-t, \sqrt{2}t\rangle = \langle x,y,z\rangle}$$

$1 = z = \sqrt{2}t \to t = \frac{1}{\sqrt{2}} \to x = 1 + 1/\sqrt{2},\ y = 1 - 1/\sqrt{2}$ !

$\boxed{\text{Point: } (1+1/\sqrt{2}, 1-1/\sqrt{2}, 1)}$

e) $$K = \frac{|r'\times r''|}{|r'|^3} = \frac{\sqrt{2}(e^t+e^{-t})}{(e^t+e^{-t})^3} = \boxed{\frac{\sqrt{2}}{(e^t+e^{-t})^2}}$$

$$\rho = 1/K = \boxed{(e^t+e^{-t})^2/\sqrt{2}}$$

f) $$\hat{N} = \hat{B}\times\hat{T}$$

$$= \frac{1}{(e^t+e^{-t})^2}\begin{vmatrix} i & j & k\\ -e^{-t} & e^t & \sqrt{2}\\ e^t & -e^{-t} & \sqrt{2}\end{vmatrix}$$

$$= \frac{1}{(e^t+e^{-t})^2}\langle \sqrt{2}e^t+\sqrt{2}e^{-t}, \sqrt{2}e^t+\sqrt{2}e^{-t}, \underbrace{e^{-2t}-e^{2t}}_{(e^{-t}-e^t)(e^{-t}+e^t)}\rangle$$

$$= \boxed{\frac{1}{e^t+e^{-t}}\langle \sqrt{2}, \sqrt{2}, e^{-t}-e^t\rangle}$$

g) $$L = \int_{-1}^{1}|\vec{r}\,'(t)|\,dt = \int_{-1}^{1} e^t+e^{-t}\,dt = e^t - e^{-t}\Big|_{-1}^{1}$$

$$= e-e^{-1}-(e^{-1}-e^1) = \boxed{2(e-e^{-1}) \approx 4.7008}$$

h) $$a_T(0) = \hat{T}(0)\cdot\vec{r}\,''(0) = \tfrac{1}{2}\langle 1,-1,\sqrt{2}\rangle\cdot\langle 1,1,0\rangle = \boxed{0}$$

$$a_N(0) = \hat{N}(0)\cdot\vec{r}\,''(0) = \tfrac{1}{2}\langle \sqrt{2},\sqrt{2},0\rangle\cdot\langle 1,1,0\rangle$$

$$= \tfrac{1}{2}(\sqrt{2}+\sqrt{2}) = \boxed{\sqrt{2}}$$

duh! yeah! $0^2+(\sqrt{2})^2 = (\sqrt{2})^2$ !

i) $$\vec{c}(0) = \vec{r}(0) + \rho(0)\,\vec{N}(0)$$

$$= \langle 1,1,0\rangle + \frac{(1+1)^2}{\sqrt{2}}\,\frac{1}{2}\langle \sqrt{2},\sqrt{2},\overset{=0}{1-1}\rangle$$

$$= \langle 1,1,0\rangle + \langle 2,2,0\rangle = \langle 3,3,0\rangle \checkmark$$